

## Uncertainty Analysis Practice - answers

Show work in this column	Calculated value	Absolute uncertainty $y$	Relative uncertainty (show as %)
$q_{\text{tot}} = -(q_{\text{cal}} + q_{\text{surr}})$ $= -(4.345 + 3.45) = -7.795 \text{ kJ}$ but we're adding and 3.45 only has precision to 100 <sup>th</sup> s place, so we round to 100 <sup>th</sup> s place $q_{\text{cal}} = 4.345 \pm 0.003 \text{ kJ}$ $q_{\text{surr}} = 3.45 \pm 0.05 \text{ kJ}$ For a sum, $AU = \Sigma AU_s = 0.003 + 0.05 = 0.053 \text{ kJ}$ but we round to 1 sf, so <u><math>AU = 0.05 \text{ kJ}</math></u> . For a sum, we have to calculate RU from the calculated AU (before we rounded), so $RU = AU/ q_{\text{tot}}  = (0.053 \text{ kJ})/(7.795 \text{ kJ}) = 0.0068$ , so, rounded to 1 sf, <u><math>RU = 0.7\%</math></u>	$q_{\text{tot}} = -7.80 \text{ kJ}$ $q_{\text{tot}}$ and AU have the same precision, so we don't need to worry about rounding to make them agree.	0.05 kJ	0.7%
$E = mad = (1.0)(170.0)(10.00) \text{ J} = 1700.00 \text{ J}$ but we're multiplying and m only has 2 sig figs, so we round to 2 sig figs $m = 1.0 \pm 0.5 \text{ kg}$ $a = 170.0 \pm 0.1 \text{ m/s}^2$ $d = 10.00 \pm 0.01 \text{ m}$ For a product, $RU = \Sigma RUs = (.5)/(1.0) + (.1)/(170) + (.01)/(10) = 0.502$ but we round all uncertainties to 1 sf, so <u><math>RU = 0.5 = 50\%</math></u> . Note that this is basically coming entirely from m, which has 50% uncertainty. For a product, we have to calculate AU from the calculated RU (before we rounded), so $AU = RU \times  E  = (0.502)(1700 \text{ J}) = 853$ , so, rounded to 1 sf, <u><math>AU = 9 \times 10^2 \text{ J}</math></u>	$E = 1.7 \times 10^2 \text{ J}$ Again, E and AU have the same precision, so we don't need to worry about rounding to make them agree.	$9 \times 10^2 \text{ J}$	50%
$C = f^2 = 5^2 = 25$ but we're multiplying and	$C = 2 \times 10$	1x10	40%

$f$ only has 1 sig fig, so we round to 1 sig fig. $C = 2 \times 10$ (an exact 5 rounds to the nearest even number) $f = 5 \pm 1$ For a product, $RU = \Sigma RUs = 1/5 + 1/5 = 2/5 = 0.4$ to 1 sf, so <u><math>RU = 40\%</math></u> . Notice that $f^2$ has twice the uncertainty of $f$ . For a product, we have to calculate AU from the calculated RU (use un-rounded numbers), so $AU = RU \times  f  = (0.4)(25) = 10$ 1 sf, <u><math>AU = 1 \times 10</math></u>	Again, E and AU have the same precision, so we don't need to worry about rounding to make them agree.		
$X = H + I - J = 14.5 + 76.5 - 4.0 = 87.0$ and since we're adding and subtracting, and all numbers are precise to the tenths place, we round to the tenths place $H = 14.5 \pm 0.6$ $I = 76.5 \pm 0.4$ $J = 4.0 \pm 0.3$ For adding or subtracting, $AU = \Sigma AU_s = 0.6 + 0.4 + 0.3 = 1.3$ but we round to 1 sf, so <u><math>AU = 1</math></u> . For addition/subtraction, we have to calculate RU from the calculated AU (un-rounded), so $RU = AU/ X  = (1.3)/(87.0) = 0.0149$ , so, rounded to 1 sf, <u><math>RU = 0.01 = 1\%</math></u> . Don't round too early! If you had rounded 0.0149 to 0.015, you would have then rounded to 0.02.	$X = 87$ In this case, the precision of X and AU differed, so we had to round both to match the least precise, which was AU, which was precise to the 1s place.	1	1%