WORKSHOP 5

UNCERTAINTY IS AN UNAVOIDABLE PART OF LIFE

Recall that you studied statistical uncertainty in Workshop 2, during lab 2. In that exercise, we used standard deviation as a measure of uncertainty. This is a convenient method of determining uncertainty when you have made repeated measurements. However, if you only make one measurement, the standard deviation is meaningless. When you make single measurements, uncertainty can be estimated based on your equipment and technique. In addition, if you have to perform calculations with measured values, the uncertainty of your final answer can be determined by a technique called "propagation of error."

Remember, also, that when you know the uncertainty in a number, the significant figures should be determined as follows: 1) Round the uncertainty to one significant figure. 2) Round the value to the same digit (e.g. the tens place) as the uncertainty.

Uncertainty in measurements

Uncertainty can also occur due to the inherent imprecision of measuring instruments. What would you do if a recipe called for 1.453 tablespoons of sugar? Do you have any cooking gadgets that can measure something that precisely? Probably not, so that measurement will be less precise (more uncertain) than the recipe requires!

Similarly, instruments in chemistry laboratory have differing levels of precision with which they can measure substances, and the level of precision you need influences which piece of glassware you use. The last digit that you read off of an instrument or piece of glassware is uncertain, but all other digits are precise. For example, if your glassware has markings every 0.5 ml, you would be estimating the final number in between the ½ ml markings.

PART A) "Reading between the lines: recording measured numbers":

Obtain a 50 mL beaker, a 10.0 mL graduated cylinder, and a 50.00 mL volumetric flask.

Questions for Discussion:

1) Would you be able to measure 1.43 mL with a 50 mL beaker? Why or why not?
2) Would you be able to measure 1.43 mL with a 10.0 mL graduated cylinder? Why or why not?
3) What is the difference between a 50 mL beaker and a 50.00 mL volumetric flask? How is this difference reflected in the way their volumes are written (i.e. 50 vs. 50.00 mL?)

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PART B) "Reading between the lines: recording measured numbers":

There are 4 containers in front of you:

- 25.0 mL graduated cylinder
- 50 mL beaker
- 100.0 mL graduated cylinder
- 125 mL Erlenmyer flask

1) In groups of two, determine how much liquid is in each container. Recall that the last digit you record should be a guess, and that all other digits should be certain.
2) Record your answers on the board, indicating correct significant figures.
3) Consider the Questions for Discussion, below.
4) Discuss your answers as a class.

Questions for Discussion:

1) Which of the four pieces of glassware was the least precise? Which was the most precise?
2) Suppose you wanted to design a container that was more precise than any of the ones you just used. What features might it have?
3) How does estimation play a role in making measurements?
4) If all four containers had exactly the same amount of liquid, would we have to report them differently? Why?
5) Can you think of a real-life application where reporting precision correctly greatly matters? There are many!

PART C) Absolute vs. relative uncertainty:

You're collecting census data for both the town of Wellesley and Wellesley College. Suppose you gather the following two statistics based on some imprecise measurements:

Population of Town of Wellesley: 26,000 ± 1000 people
Population of Wellesley College: 2400 ± 200 people

1) Which measurement is more certain?
2) As a class, discuss absolute and relative uncertainty.
3) Go back to the measurements in part A and calculate the relative uncertainty for each measurement now. Which has a smaller relative uncertainty?

When writing uncertainty, we write: **VALUE ± AU**, where AU = absolute uncertainty.

Relative uncertainty = **RU =** ___________________

Questions for Discussion:

1) Why is absolute uncertainty useful?
2) Why is relative uncertainty useful?
3) In what kind of situations might one be more important than the other?

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PART D) Propagation of uncertainty: addition and subtraction

Does 1+ 1 = 2? Maybe, but it depends on your uncertainty!

What if you weigh out 1 gram of baking soda 100 times, put all of the baking soda together, and weigh the total. Will the mass be exactly 100 grams? Perhaps.

A scale that is only precise to the ones place will read 1 g whether you have 0.7 g or 1.3 g. If your sample was actually 1.3 g each time, your total after 100 measurements would be 130 grams! Uncertainty can make a big difference!

Several experiments were performed, in which several aliquots of water were measured and added together, each experiment using different glassware or balances. The total after many measurements was recorded, and you will be given the data to analyze.

Read about each experiment, and look over the data given to you by your instructor.

Questions for Discussion:

1) Was the measurement of the sum exactly equal to the sum of the measurements in each case? Why or why not?
2) Which experiment did you expect to give the best results? Which did you expect to be worst? Why?
3) The top loading balance give masses good to ± 0.01 g. Suppose you make 4 individual measurements and you want to know what the mass of the sum is, so you add the four numbers together. What is the maximum amount by which this sum could be off by? How did you arrive at this number?
4) Can you come up with a general rule for determining the uncertainty of a value when it's obtained by adding or subtracting measured values?
5) Go back and use this rule to determine the uncertainties in each sum above. Does the measured value fall within the uncertainty range in each case?

Discuss as a class. You just devised a rule for determining the *maximum* uncertainty in a value obtained by adding or subtracting uncertain values. **This is the rule we will be using in CHEM 105 when propagating uncertainty for addition and subtraction:**

$$\text{If } S = A + B - C$$

$$\text{then } AU_S = \underline{\hspace{6cm}}$$

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PART E) Propagation of uncertainty: multiplication and division.

You are looking after a patient that needs to get a precise concentration of a particular drug. To make this solution, you need to dissolve 0.1000 g of substance A in 20.00 mL of water. This solution can be made in many ways, some of which will lead to a more precise concentration of final drug than others.

Several experiments were performed, in which the solution was prepared using different balances for measuring mass and different glassware for measuring the water volume. You will be given the data to analyze.

Read about each experiment, and look over the data given to you by your instructor.

Questions for Discussion:

1) Assume drug A has a molar mass of 588.24 g/mol. Calculate the concentration of drug solution you wanted to make, assuming 20.00 mL of total solution. Note that this calculation involves multiplication and division, not addition and subtraction.
2) Was the concentration exactly the same for each experiment?
3) Which method would you expect to be the most precise? The least precise?
4) Which method *was* the mast precise? The least precise?
5) Which method do you think took the longest?
6) If the patient can actually tolerate a wide range of concentrations, which method might a company adopt?
7) Relate this statement to uncertainty in this experiment: "You're only as strong as the weakest link"
8) After class discussion, go back and calculate the uncertainties in each of the concentrations reported in the data table containing the class concentration data. Do the actual concentrations fall within the uncertainties?
9) Why are absolute uncertainties used to propagate error when adding and subtracting, and why are relative uncertainties used to propagate error when multiplying and dividing? (Think about units as well as mathematical functions.)

How could you assign uncertainty to the concentration you calculated in question A? This value required multiplication/division of two measured quantities (assume there is no uncertainty in the molar mass). Can you come up with a rule to assign uncertainty to a value obtained by multiplying or dividing two numbers?

Discuss as a class. **This is the rule we will be using in CHEM 105 when propagating uncertainty for multiplication and division:**

$$If\ S = AB/C$$

$$then\ RU_S = \underline{\hspace{6cm}}$$

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Optional Material:

The rules we devised above represent the ***maximum*** *uncertainty. Sometimes, this is too conservative. What might be a better way to quantify uncertainty is to report the "likely" range of such a value. The following formulas are used to determine a more realistic "probable" range of a value acquired by adding or subtracting two measured quantities.* ***You will use these formulas in Chem 205:***

$$\text{If } S = A + B - C$$

$$\text{then } AU_S = \sqrt{AU_A^2 + AU_B^2 + AU_C^2}$$

$$\text{If } S = AB/C$$

$$\text{then } RU_S = \sqrt{RU_A^2 + RU_B^2 + RU_C^2}$$