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Lecture 3: The Solow Growth Model

I. OVERVIEW

- Economic growth is by far the most important topic in economics. I can try to describe to you how important it is but my words would pale in comparison to the words of the Nobel Prize winning economist Robert Lucas who stated the following in a 1998 paper.

  > *Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not what is it about the "nature of India" that makes it so. The consequences for human welfare involved in questions like these are simply staggering. Once one starts to think about them, it is hard to think about anything else*

- Another way to emphasize the importance of this is to use the following fact: If an economy grows at g% we can show that it doubles roughly every $70/g$ years. So if an economy grows at 2% it will double every 35 years and if it grows at 7% it will double every 10 years.

- Using this rule we can illustrate the power of economic growth using an example.
  - From 1960-1995 South Korea grew at 6% a year. This means that their economy doubled every 11.5 years on average. Over a period of 35 years income would double 3 times: an 8 fold increase.
  - Had they grown at 5% a year they would have doubled their income every 14 years. Over a 35 year period income would double 2.5 times: increase $2^{2.5} = 5.65$ fold.
  - Had they grown at 4% a year they would have doubled their income every 17.5 years. Over a 35 year period their income would double 2 times: increase 4 fold.
  - Therefore, even a 1%-2% change in growth rates can lead to large changes in income. If we know what drives growth, we can have significant positive impacts on the living standards of people all over the world.

II. A MODEL OF ECONOMIC GROWTH

- What constitutes a good model of economic growth? Let's first use our intuition and identify some important determinants of economic growth.

Capital	Geography	Labor
Trade	Human Capital	Corruption
Technology	Institutions	Infrastructure

- Myriad factors can affect economic growth in a country, but we can broadly conclude that growth is basically determined by its ability to produce goods and services. How does it produce goods and services? It uses two important inputs: labor and capital and combines them with know-how to produce output; economists refer to the knowledge about putting inputs together as technology.

- Labor and capital are called inputs. Technology enables us to put inputs together in order to make output. Silicon and metal to make computer chips. Rubber and chrome to make tires etc.
- So we can write down a production function that describes how labor, capital and technology get transformed into output. $Y = F(K, L, T)$ where K is capital, L is labor and T is technology.
- If output is produced using these 3 factors, then growth in output must come from growth in either K, L or T. Basically, an economy can start producing more output if it has more workers, more machines, or better ways of putting together machines and workers.
- A good model should enable us to understand the importance of most, if not all, of these variables for economic growth. It should also help us understand less intuitive questions such as whether growth will increase permanently or temporarily in response to changes in the capital stock, for example.
- Alternatively, it should help us understand whether an economy will invest in more capital when it has better technology or whether it will continue to use the same amount of capital and make better use of it.
- We will use the most famous model of economic growth pioneered by Robert Solow, who won a Nobel Prize. The model has many simplifying assumptions, yet it is a useful start for our analysis of growth. In my opinion, the Solow model is the best economic model. It is simple, yet yields powerful, intuitive conclusions. It has very clear simplifying assumptions that can be relaxed to make the model more complex.
- The Solow Model consists of two equations: a production function and a capital accumulation equation.

III. The Solow Model

The Production Function

- We make the following assumptions about the production function.

  1. There are only 2 inputs, which we will denote by K (capital) and L (labor), and 1 output good, which we will denote by Y.
  2. The production function exhibits Constant Returns to Scale (i.e. doubling K and L doubles Y)
  3. The production function exhibits diminishing return to labor and capital (increases in one input, holding the other constant, yield fewer and fewer additional units of output).
  4. We will assume that the growth rate of the labor force is given exogenously (from outside the model). Let the growth rate of the labor force $\frac{\dot{L}}{L} = n$.

- A production function that works well is of the Cobb-Douglas form $Y = K^{\alpha}L^{1-\alpha}$, where α (which is positive and < 1) is the share of output produced by capital; e.g. if $\alpha = 0.3$ then 30% of output is produced by capital and 70% by labor.

- By taking logs and differentiating we can obtain the following:

$$\begin{aligned} Y &= K^{\alpha}L^{1-\alpha} \Rightarrow \ln(Y) = \alpha\ln(K) + (1-\alpha)\ln(L) \\ \Rightarrow \frac{\dot{Y}}{Y} &= \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} \\ &= \alpha\frac{\dot{K}}{K} + (1-\alpha)n \end{aligned}$$

- From this we can see that in order to understand the growth rate of Y we need to understand the growth rate of K, which is determined endogenously (within the model).
- This leads us to the second equation in the Solow model: the capital accumulation equation which describes how the capital stock evolves over time.

The Capital Accumulation Equation

- The second equation of the model is

$$\dot{K} = sY - \delta K$$

- In this equation s is the saving rate: a fraction of every unit of output is saved and δ is the depreciation rate: a fraction of every unit of capital is worn out. Both s and δ are exogenous to the model.
- Intuition for this equation lies in the national income accounting identity for a closed economy (where $X - M = 0$)

$$Y = C + I + G$$

- We can then rearrange to get $(Y - C - T) + (T - G) = I$ where $T =$ total tax revenue. This identity states that
  Private Savings + Gov't Savings = Gross Investment
  or equivalently that
  Total Savings = Gross Investment.
- In a closed economy, gross investment (new additions to the capital stock) is constrained to be equal to the amount of savings in the economy.
- However, some of these new additions to the capital stock merely replenish worn out portions of the existing capital stock. This amount is known as "replacement investment".
- Therefore, net investment, the change in the capital stock, is the difference between gross investment and replacement investment.
- In the capital accumulation equation $\dot{K} = sY - \delta K$ then

$$\begin{aligned} sY &= \text{Total Savings} = \text{Gross Investment} \\ \delta K &= \text{Replacement Investment} \\ sY - \delta K &= \text{Net Investment} \end{aligned}$$

- When
  - total savings exceeds replacement investment($sY > \delta K$), the capital stock increases ($\dot{K} > 0$).
  - total savings is less than replacement investment($sY < \delta K$), the capital stock decreases ($\dot{K} < 0$).
  - total savings equals replacement investment ($sY = \delta K$) the capital stock does not change ($\dot{K} = 0$)
- Note: δK is also referred to as the break-even level of investment: the amount of investment necessary to leave the capital stock unchanged.