Fall Semester '05-'06
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Lecture 6: Assessing the Basic Solow Model

I. OVERVIEW

- In the last class we looked at a couple of comparative statics exercises. The first was the impact of a change in the saving rate on capital and output in the economy. We showed that the change in the saving rate did not have a long run impact on the growth rate of output.
- However, there was a short run increase in the growth rate, and as a result the economy reached a higher level of steady state output. A change in the saving rate is therefore said to have a **level effect** on output but NOT have a **growth effect**; i.e. it affects the steady state level of output but not the long run growth rate of output.
- The second was the impact of a change in the population growth rate on capital and output in the economy. We showed that the change in the population growth rate, while raising both the level and the growth rate of total output in the economy, left the level of per-capita output lower than it would have been in the absence of the increased population growth.
- Today's class looks at some of the mathematical properties of the steady state to understand how economic changes affect the endogenous variables of the model.

II. COMPARING STEADY STATES USING ALGEBRA

- In economics, we typically use calculus to do comparative statics exercises. Given an economic model, we would solve for the endogenous variables as functions of the exogenous variables and the parameters, then take derivatives (or partial derivatives) to show how the endogenous variable will change when an exogenous variable or a parameter changes.
- However, thus far with the Solow model we have used diagrams instead of calculus to perform the various comparative statics exercises. The reason for doing so is that the Solow model is a dynamic model whose behavior is governed by a differential equation, the now familiar $\dot{k} = sy - (n + \delta)k$ equation.
- Since differential equations is not a pre-requisite for this course, we won't solve this equation for the value of k at a given point in time. We can however, avoid the complications of the differential equation and use calculus if we restrict our focus to the steady state. Why? Because at the steady state $\dot{k} = 0$, thus eliminating the troublesome left-hand side of the above equation.
- The capital accumulation equation can now be written as (using * to denote steady-state values) $0 = sy^* - (n + \delta)k^*$. This simplifies to

$$
\begin{aligned}
(n+\delta)k^* &= sy^* \equiv sk^{*\alpha} \\
\Rightarrow \frac{n+\delta}{s} &= k^{*\alpha-1} \\
\Rightarrow \frac{s}{n+\delta} &= k^{*1-\alpha} \\
\Rightarrow k^* &= \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

- Using the fact that $y = k^\alpha$ we can then show that

$$
y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}
$$

- From these expressions we can see that the Solow model predicts that increases in the saving rate will raise steady state income per-capita since $\frac{dy^*}{ds} > 0$
- The model also predicts that increases in the depreciation rate and the population growth rate will reduce steady state income per-capita, i.e. $\frac{dy^*}{dn} < 0$ and $\frac{dy^*}{d\delta} < 0$
- In addition, using the fact that $k = \frac{K}{L}$ and $y = \frac{Y}{L}$ we can show that

$$
K_t^* = L_t\left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}} \text{ and } Y_t^* = L_t\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}
$$

- From these expressions we can see that the Solow model predicts that increases in the saving rate will raise steady state aggregate income since $\frac{dY^*}{ds} > 0$
- Similarly, increases in the depreciation rate will reduce steady state aggregate income since $\frac{dY^*}{d\delta} < 0$
- Third, increases in the size of the population (labor force) will increase aggregate income $\frac{dY^*}{dL} > 0$
- Finally, an increase in the growth rate of the population will actually increase aggregate income $\frac{dY^*}{dn} > 0$. This may seem inconsistent with the above expression given that n appears in the denominator but don't forget that n is also part of L_t, i.e. $L_t = L_0e^{nt}$. Since exponential functions grow faster than linear functions the L_t term increases faster than the $\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$ term decreases, implying that $\frac{dY^*}{dn} > 0$
- We can also take a look at what the algebraic solutions say about steady-state growth. Since the right-hand sides of the expressions $k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$ and $y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$ are all parameters, we can see that per-capita output and capital is constant in steady state.
- Similarly, we can see that in steady state, K and Y are not constant, they are growing over time (hence the subscript t) because the population is growing. In other words steady state aggregate output and capital will grow at the same rate as the population.

III. ASSESSING THE PREDICTIONS OF THE SOLOW MODEL

- Now that we have an idea about the properties of the Solow model, let's assess the intuitiveness of the predictions of the Solow model.

1. Why are some countries rich and others poor?

- The Solow model predicts that countries with high investment (saving) rates, low population growth rates and low rates of depreciation are likely to have more output per worker.
- The model also predicts that all else being equal, i.e. if the investment rates, population growth rates and depreciation rates are the same, countries with a larger population are likely to be have larger economies.
- The predictions of the model are therefore quite sensible in answering this question.

2. Why do some countries grow faster than others?

- The Solow model has very little to say about difference in per capita economic growth across countries - in the long run all countries have a steady state output per capita growth rate of zero - this is an unsatisfying answer and a weakness of the model.
- The Solow model has a little bit more to say about the growth rate of total output: it is equal to the rate of population growth. This seems to say that countries with high population growth rates should grow faster.
- This is not a very intuitive answer; it is more palatable if you think of n as the growth rate of the labor force. But the model does not provide us the ability to distinguish the labor force from the population.

3. How can economies exhibit sustained economic growth?

- In per capita terms, the Solow model does not allow for sustained growth so it may not be helpful for describing the behavior of the U.S economy for example. We therefore need to expand the Solow model, by adding technology, in order to improve its predictions.
- In addition to being intuitive, predictions of a model should also be consistent with reality, i.e. be empirically sound. Is it true that

  a) countries with high investment (saving) rates, low population growth rates and low rates of depreciation are likely to be richer in per-capita terms?
  b) among countries with similar population growth rates, saving rates and depreciation rates, countries with larger populations have larger economies?
  c) all countries have the same steady state growth rate of output per worker and capital per worker; namely, zero?
  d) countries with higher rates of population growth will be the ones whose economies will grow faster in the aggregate?

- In class, we will also examine the viability of these claims using data for the last few decades for a range of countries.