Fall Semester '05-'06
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Lecture 7: Growth Away From Steady State

I. OVERVIEW

- In the last lecture we examined some properties of the steady state. In the lecture before that, we drew the graphs for the time paths of k, y, K and Y in various comparative statics exercises.
- One of the issues left unresolved was the shape of the graph during the transition period. Essentially, we need to have a better explanation of the growth rates of these variables away from steady state. Today's lecture takes a closer look at the behavior of economies away from steady state.
- This is important because many of the interesting growth stories in the world are countries in transition. Consider China whose economy grows at 10% a year. China couldn't be in steady state because at a 10% growth rate their income would double every 7 years and be 16,384 times larger than today in a hundred years time. So to understand the behavior of economic growth in China, we need to dig deeper into the Solow model.

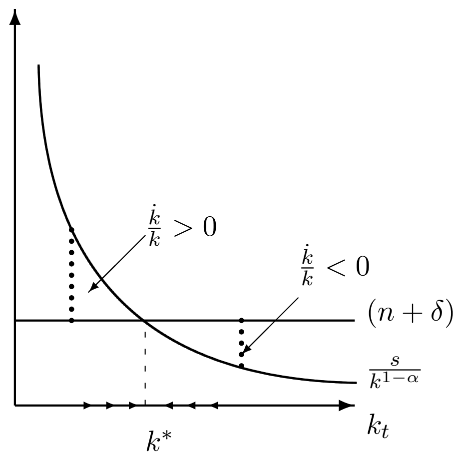
II. GROWTH RATES AWAY FROM STEADY STATE

- Let's take another look at what the Solow model has to say about growth rates of y and Y away from steady state. In order to do this we look at a modified Solow diagram obtained from the capital accumulation equation.

$$\begin{aligned} \dot{k} &= sy - (n+\delta)k \Rightarrow \frac{\dot{k}}{k} = \frac{sy}{k} - (n+\delta) \\ \frac{\dot{k}}{k} &= \frac{sk^{\alpha}}{k} - (n+\delta) \\ \frac{\dot{k}}{k} &= \frac{s}{k^{1-\alpha}} - (n+\delta) \end{aligned}$$

- The graph for $\frac{s}{k^{1-\alpha}}$ is downward sloping since an increase in k raises the value of the denominator. To put it in terms of derivatives, we can show that $\frac{d(sk^{\alpha-1})}{dk} = s(\alpha-1)k^{\alpha-2} < 0$
- Furthermore, we can calculate the 2nd derivative as positive because $\frac{d^2(sk^{\alpha-1})}{dk^2} = s(\alpha-1)(\alpha-2)k^{\alpha-3} > 0$. In other words, the shape of this curve is convex.
- You can also show that $\lim_{k\to 0} \frac{s}{k^{1-\alpha}} = \infty$ and $\lim_{k\to\infty} \frac{s}{k^{1-\alpha}} = 0$
- $(n+\delta)$ is of course just a horizontal line that doesn't change with $\tilde{k}$

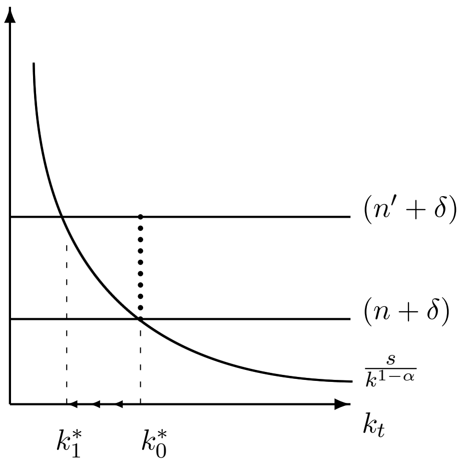
- The picture below graphs these two lines. The difference between the $\frac{s}{k^{1-\alpha}}$ and the $(n+\delta)$ lines (indicated by the dotted lines in the figure below) shows the growth rate of k, i.e. gives us the value of $\frac{\dot{k}}{k}$



- In this diagram, for any level of capital per worker (k), the gap between the downward sloping curve and the horizontal line is the growth rate of capital per worker $\frac{\dot{k}}{k}$.
- We can conclude the following:
  1. At the steady state, the point at which the two lines cross, $\frac{\dot{k}}{k} = 0$
  2. As the economy approaches steady state from below, $\frac{\dot{k}}{k} > 0$ but getting closer to 0.
  3. As the economy approaches steady state from above, $\frac{\dot{k}}{k} < 0$ but getting closer to 0
- This is what gives us the concave and convex shapes in drawing the time paths of variables. When moving from a lower (constant) steady state to a higher (constant) steady state, we should see a positive growth rate that is gradually approaching zero, i.e. a concave shape.
- Conversely when moving from a higher (constant) steady state to a lower (constant) steady state, we should see a negative growth rate that is gradually approaching zero, i.e. a convex shape.
- Using the fact that $y = k^\alpha$ we can also derive the following results for the growth rate of y
  1. At the steady state, the point at which the two lines cross, $\frac{\dot{y}}{y} = 0$
  2. As the economy approaches steady state from below, $\frac{\dot{y}}{y} > 0$ but getting closer to 0.
  3. As the economy approaches steady state from above, $\frac{\dot{y}}{y} < 0$ but getting closer to 0
- Basically, the Solow model predicts that the further below steady state an economy is, the faster output per worker will grow and the further above steady state an economy is, the slower output per worker will grow.
- Finally, using the fact that $\frac{\dot{Y}}{Y} = \frac{\dot{y}}{y} + n$ we can also derive the following prediction: the further below steady state an economy is, the faster total output will grow and the further above steady state an economy is, the slower total output will grow.

III. MORE SUBTLETIES REGARDING GROWTH RATES AWAY FROM STEADY STATE (THIS IS OPTIONAL - ONLY FOR THE REALLY CURIOUS)

- When we discussed comparative static exercises using the Solow model, one of the interesting cases occurred when the population growth rate increased from n to n'.
- We showed that in the initial steady state, total output was growing at a rate n and in the new steady state total output was growing at a rate n'. In the interim, our calculations indicated that the growth rate would be $< n'$ but we could not say whether or not it would be $> n$.
- We can use the diagram above to show that the transition growth rate is always between the initial steady state growth rate and the new steady state growth rate.
- The picture below graphs the two lines $(n + \delta)$ - the old break even line and $(n' + \delta)$ - the new break even line.
- At the old steady state, where growth was previously zero, now growth will be < 0. Since the difference between the $\frac{s}{k^{1-\alpha}}$ and the $(n + \delta)$ lines (indicated by the dotted lines in the figure below) shows the growth rate of k, we can calculate the exact value of growth as being $\frac{s}{k_0^{*1-\alpha}} - (n' + \delta) \equiv (n + \delta) - (n' + \delta) = n - n'$



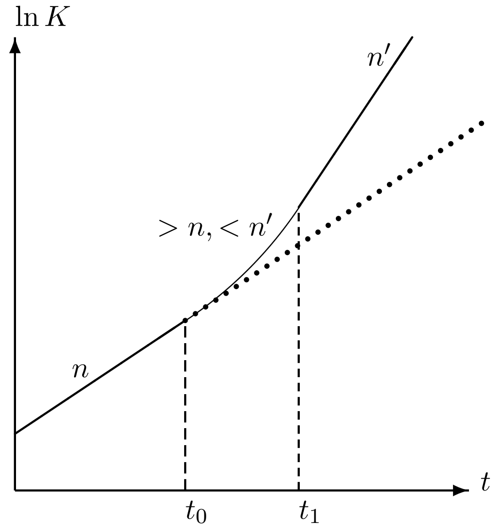
- We can summarize the growth rates of the key variables in the following table:

Time	$\frac{\dot{k}}{k}$	$\frac{\dot{L}}{L}$	$\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + \frac{\dot{L}}{L}$
Before t_0	0	n	n
At t_0	$(n - n')$	n'	n
B/w t_0 and t_1	$0 > \frac{\dot{k}}{k} > (n - n')$	n'	$n' > \frac{\dot{K}}{K} > n$
At t_1	0	n'	n'

1. At the old steady state, now $\frac{\dot{k}}{k} = n - n' < 0$. Since $\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + n'$ we have that at the old steady state $\frac{\dot{K}}{K} = n - n' + n' = n < n'$

2. As the economy approaches the new steady state from above, $(n - n') < \frac{\dot{k}}{k} < 0$ but increasing and getting closer to 0. Since $\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + n'$ we have that as we approach the new steady state $n < \frac{\dot{K}}{K} < n'$ but increasing and approaching n'

- In other words the interim growth rate of the K variable is initially n, but then gradually approaching n', giving us the shape below. It never drops below n.



IV. CONVERGENCE

- Even though we said that the Solow model did not do a good job explaining differences in steady state growth rates, it does better at explaining differences in growth rates across countries that are away from steady state.

- One early explanation for differences in growth rates across countries was the theory of **convergence**, namely, the idea that poor countries would grow faster than rich ones. The underlying economic intuition was that, because of diminishing returns, countries with a low level of k would have a higher marginal product of capital and hence attract more investment and grow faster.

- However, the empirical evidence convergence seems to show that convergence only seems to hold within industrialized economies such as the OECD countries. If you look at a larger sample of countries, then poor countries do not seem to grow faster than rich ones. (Refer to graphs shown in class)

- Those graphs suggest a modification of the theory of convergence: among countries with the same steady state level of capital per worker, poorer countries will grow faster than rich ones. More rigorous analysis can be used to show that the model also predicts that among countries that don't have the same steady state, countries that are further below their own steady state grow faster.

- This modified theory of convergence is also known as **Conditional Convergence**. Consider the sample of countries in which we failed to find a link between the level of y and the growth rate of y: we now find a stronger link between the deviation of y from steady state and the growth rate of y (Refer to class graphs)

- The difference between convergence and conditional convergence can be shown in the following example. Consider the United States and Mali. The theory of convergence says that if $y_{MALI} < y_{US}$ it must be the case that $\left(\frac{\dot{y}}{y}\right)_{MALI} > \left(\frac{\dot{y}}{y}\right)_{US}$.

- Conditional convergence says that this is not necessarily true; instead the idea is that if $\left(\frac{y}{y^*}\right)_{MALI} < \left(\frac{y}{y^*}\right)_{US}$ then $\left(\frac{\dot{y}}{y}\right)_{MALI} > \left(\frac{\dot{y}}{y}\right)_{US}$ regardless of whether Mali was poorer than the U.S.

- In other words the theory of "Convergence" says that a poorer a country is the faster it will grow. That theory does not seem to be supported in reality. The theory of "Conditional Convergence" states that the poorer an economy is *relative to its steady state* the faster output per worker will grow. In other words it is not whether a country is poor or whether a country is rich, but whether it is poor or rich relative to its steady state.

- The Solow model predicts conditional convergence instead of convergence. Therefore it does a decent job of explaining differences in growth rates across countries.