Fall Semester '05-'06
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Lecture 10: Comparative Static Analysis

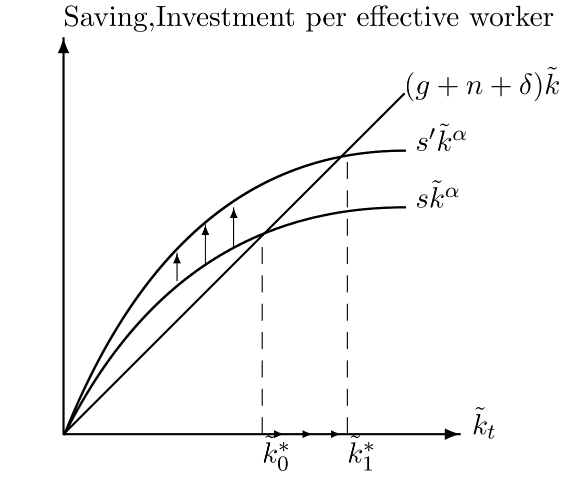
I. OVERVIEW

- In the last lecture we derived the Solow model with technology. We drew a modified version of a Solow diagram in terms of capital per effective worker and then looked at the properties of the steady state. In the steady state, capital per worker and output per worker grow at the rate of technology while capital and output grow at the rate of population + the growth rate of technology.
- Today's lecture looks at the results of some comparative static exercises using the Solow model with technology. As with the basic Solow model, we can look at the economic impact of changes in the rate of population growth, the saving rate, the rate of depreciation or the level of capital or labor in the economy.
- In addition to these changes, we can also look at changes in the growth rate of technology as well as the impact of changes in the level of technology.
- As I mentioned earlier, hopefully, adding technology to the model will preserve the sensible predictions of the basic Solow model and improve some of the less-sensible predictions of the basic model.

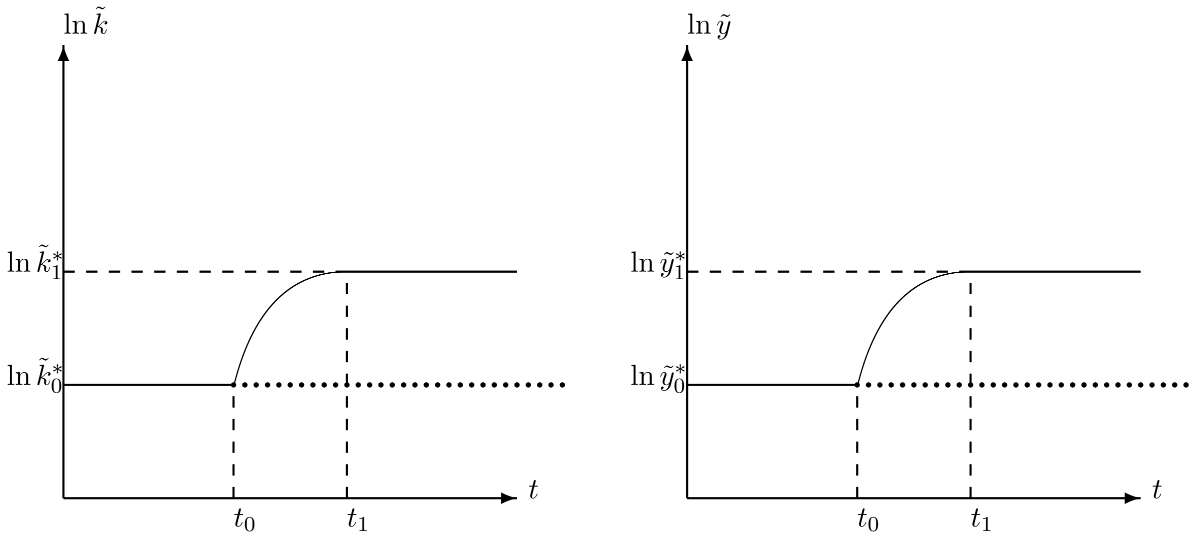
II. SAMPLE COMPARATIVE STATICS EXERCISE

An Increase in the Saving Rate

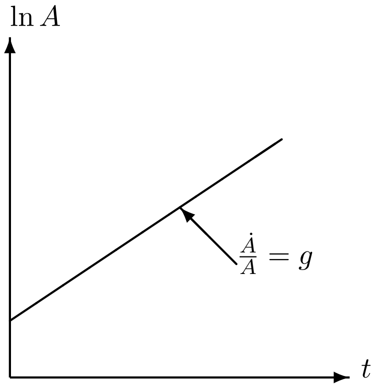
- Suppose the economy is at the steady state ($\tilde{k}_0^*$) when the saving rate increases from s to s'. For simplicity, assume that this is the only change in the economy.
- We can use the Solow diagram to illustrate the effects of the change: in the diagram the saving per worker line shifts upwards from sy to sy'. At the old steady state, $\tilde{k}_0^*$, saving per worker exceed the level of break-even investment per worker ($s'\tilde{y}_0^* > (g+n+\delta)\tilde{k}_0^*$): therefore $\tilde{k}$ increases.
- This process continues until the new steady state $\tilde{k}_1^*$ is reached. At this new steady state, the saving per worker line is once more equal to break-even investment per worker ($s'\tilde{y}_1^* = (g+n+\delta)\tilde{k}_1^*$) therefore $\tilde{k}$ does not change.
- This is illustrated in the diagram below.

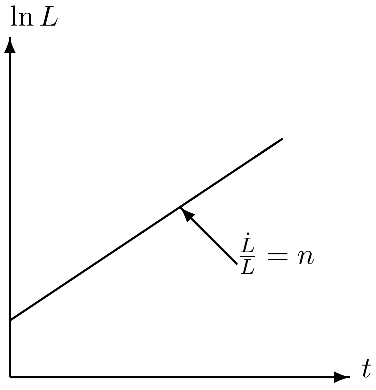


- Let t_0 be the point at which the saving rate increased and t_1 be the time at which the economy returned to steady state. We can draw the following graphs to show the path of $\tilde{k}$ and $\tilde{y}$

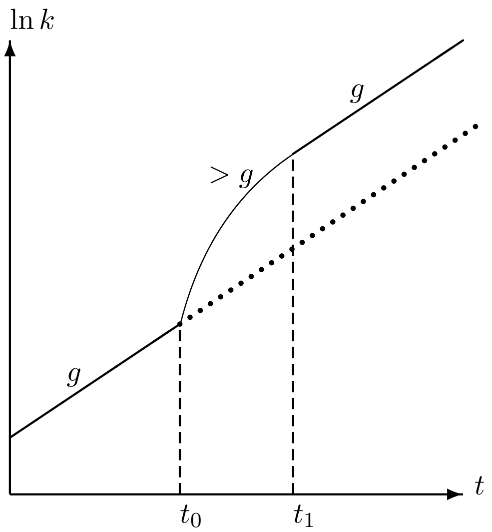


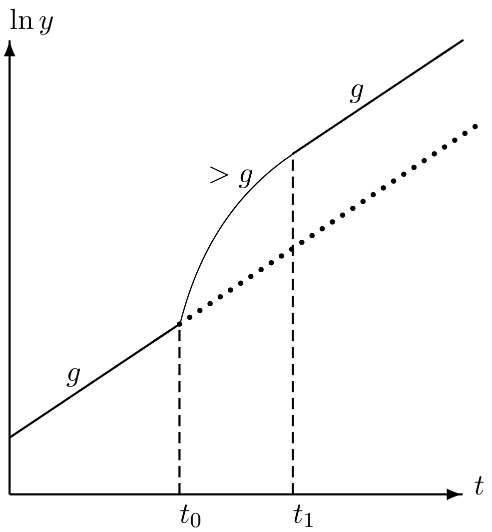
- Note: in the graph for $\tilde{k}$
  1. Before t_0 the economy is at steady state: $\tilde{k}$ is constant at $\tilde{k}_0^*$.
  2. Between t_0 and t_1 , $\tilde{k}$ is increasing over time (see Solow diagram).
  3. After t_1 the economy is back at steady state: $\tilde{k}$ is constant at $\tilde{k}_1^*$.
- NOTE: The dotted line shows the path of the variable in the absence of an increase in the saving rate.
- Since $\tilde{y} = \tilde{k}^{\alpha}$ and α is constant, the graph for $\tilde{y}$ looks exactly the same as the graph for $\tilde{k}$.
- Since nothing happened to technology or labor (they continued to grow exogenously at rate g and n respectively) the graphs for A and L can be drawn easily.



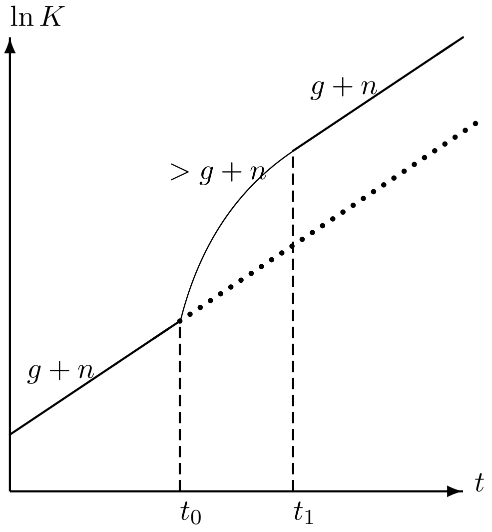


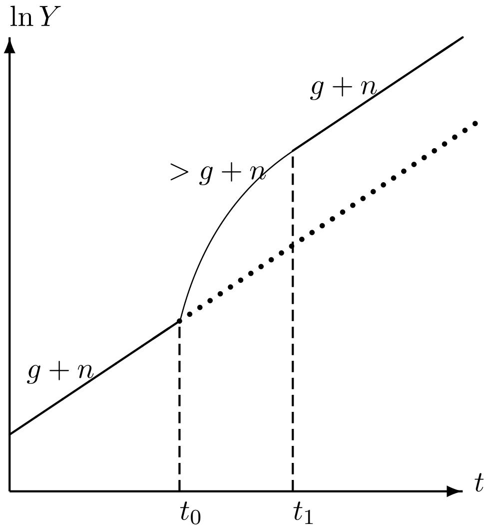
- We can use these the graphs for $\tilde{k}$ and $\tilde{y}$ along with the graph for A to deduce the behavior of k and y, using the fact that $\tilde{k} = \frac{k}{A}$ and $\tilde{y} = \frac{y}{A}$.
- Before t_0, $\tilde{k}$ is constant. Therefore if A is increasing at a rate g then k must also be increasing at a rate g in order to keep the ratio constant.
- Between t_0 and t_1, $\tilde{k}$ is increasing over time. Given that A is increasing at a rate g then k must be increasing at a rate $> g$ in order to make the ratio grow.
- After t_1 the economy is back at steady state: $\tilde{k}$ is constant. Given that A is growing at a rate g then k must also be growing at a rate g, as in the case before t_0.
- The graph for y can be derived in the same way and in this case will look identical to the graph for k.





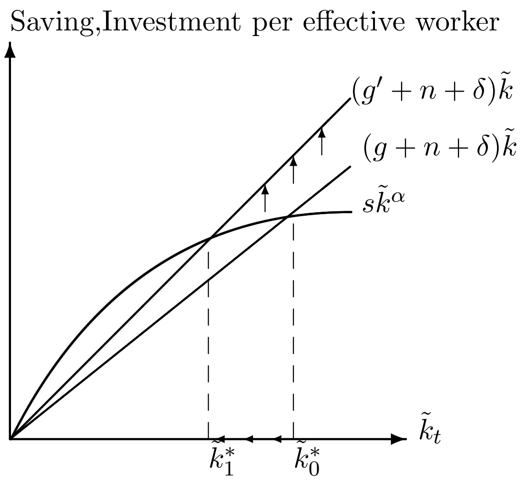
- Finally we can use the graphs for k and y, along with the graph for L to deduce the behavior of K and Y, using the fact that $k = \frac{K}{L}$ and $y = \frac{Y}{L}$.
- I will leave it up to you to check that the graphs look like the following



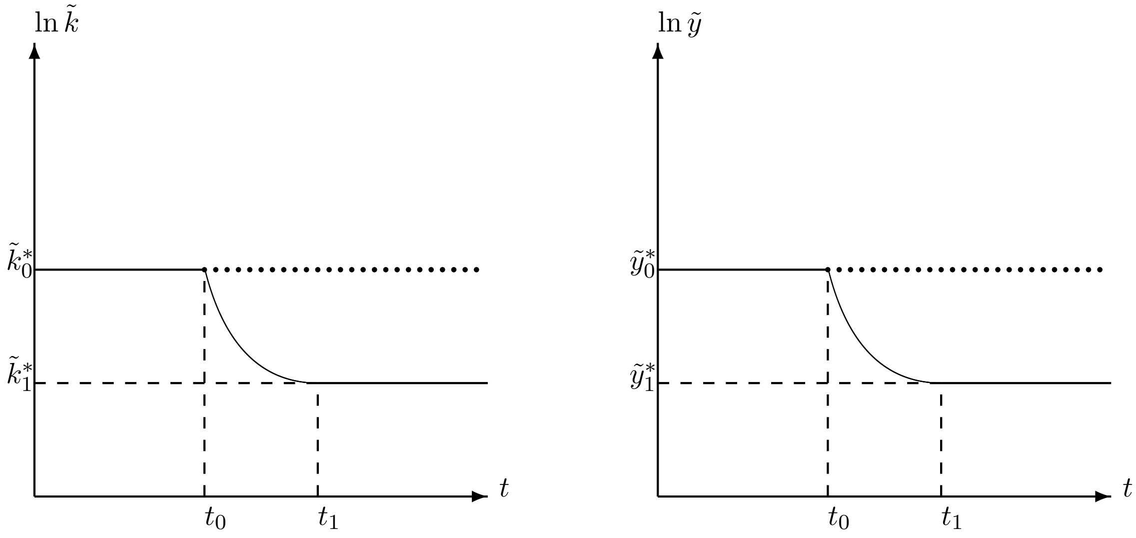


An Increase in the Growth Rate of Technology

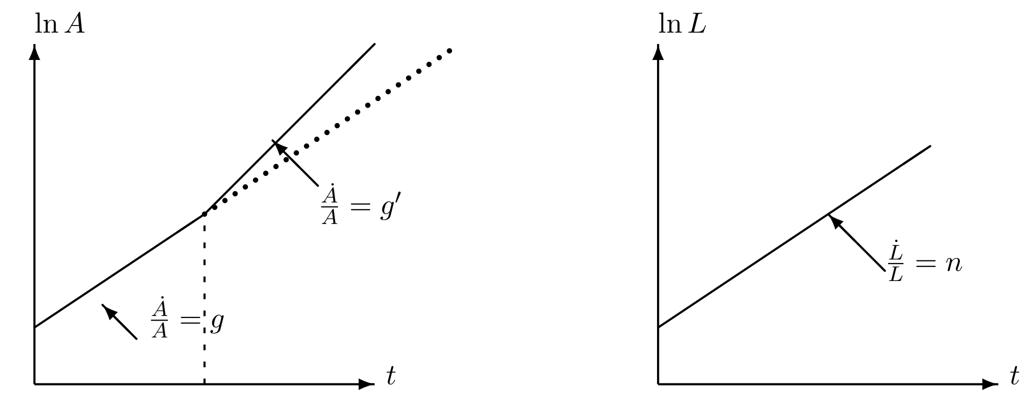
- Suppose the economy is at the steady state ($\tilde{k}_0^*$) when the growth rate of technology increases from g to g'. For simplicity, assume that this is the only change in the economy.
- Then the break-even investment per effective worker line becomes steeper. At the old steady state, saving per effective worker lies below the level of break-even investment per effective worker: therefore $\tilde{k}$ decreases.
- Why does this happen? Well, at the old steady state, the break-even requirements are higher: with better technology, more investment needs to be used to keep capital per effective worker constant. Essentially, more investment is needed to make use of the faster growing technology.
- This process continues until the new steady state ($\tilde{k}_1^*$) is reached. At this new steady state, saving per effective worker is once more equal to break-even investment per effective worker.
- The Solow diagram would look as follows:



- The dynamics of the endogenous variables $\tilde{k}$ and $\tilde{y}$ can be determined using the Solow diagram.
- The graphs for $\tilde{k}$ should then be constant at $\tilde{k}_0^*$ until t_0, at which point $\tilde{k}$ begins to decline until t_1 when it reaches its new steady state $\tilde{k}_1^*$ where $\tilde{k}$ becomes constant again.
- Since $\tilde{y} = \tilde{k}^{\alpha}$,and α has not changed, the graph for $\tilde{y}$ looks identical to the graph for k.



- The evolution of technology is more interesting here. There is an increase in the growth rate of technology at t_0 causing the slope of the curve to change from g to $g' > g$. The growth of the labor supply is constant at a rate n

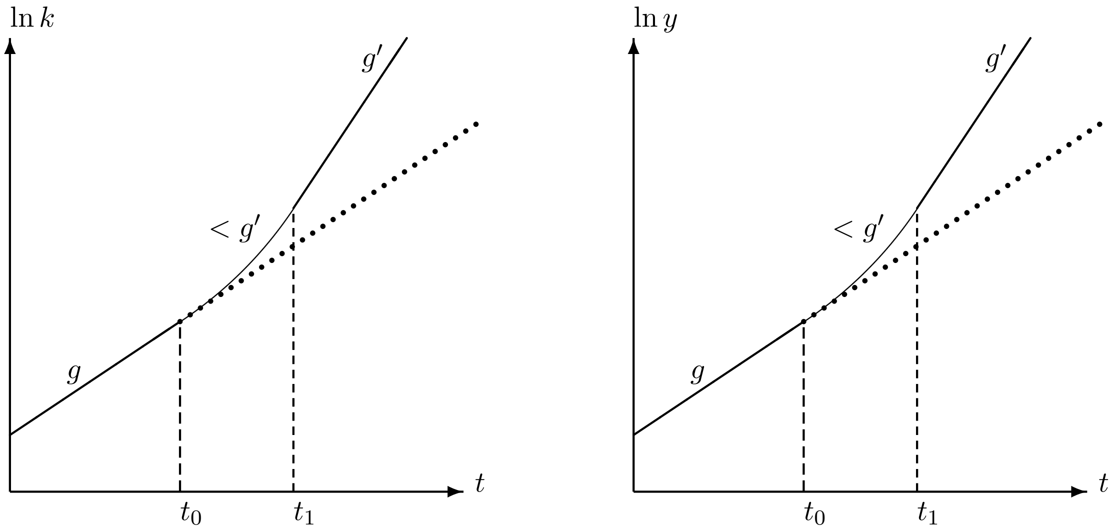


- As before, we can determine the time paths of the variables k and y from the time paths of $\tilde{k}$, $\tilde{y}$ and A.
- Consider the behavior of k where that $\tilde{k} = \frac{k}{A}$. Before t_0, $\tilde{k}$ is constant. Therefore since A is increasing at a rate g then k must also be increasing at a rate g.
- Between t_0 and t_1, $\tilde{k}$ is decreasing. Given that A is increasing at a rate at a rate $g' > g$ then k must be increasing at a rate $< g'$

- Recall once again that we can only deduce that the the growth rate of k must be $< g'$ using this technique, but as we showed in an earlier class, the growth rate is $g < \frac{\dot{k}}{k} < g'$.

- After the economy is back at steady state: $\tilde{k}$ is constant. Given that A is now growing at a rate g' then k must also be growing at a rate g', i.e. k is growing faster than it did before.

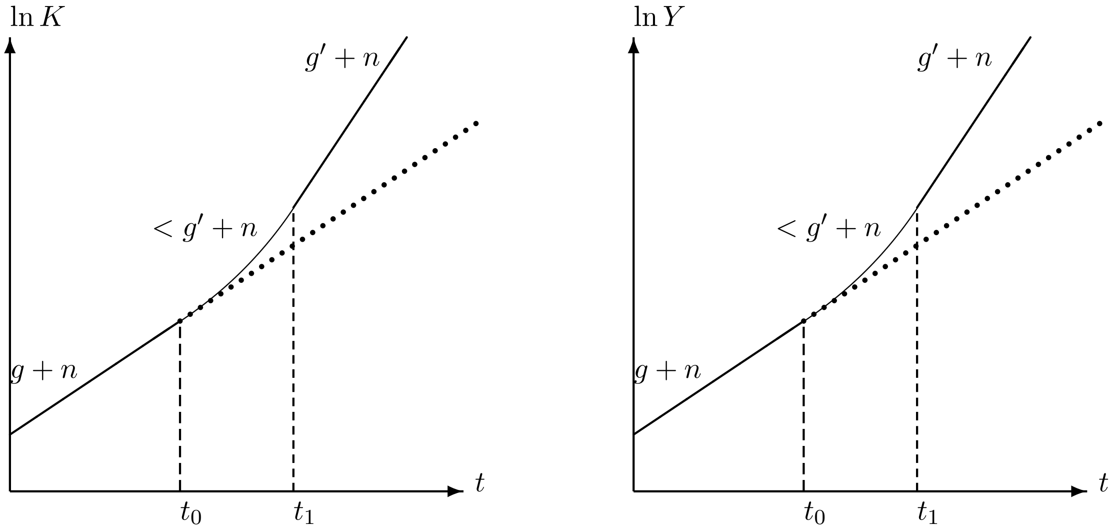
- The analysis for y will be identical since we can use the fact that $\tilde{y} = \frac{y}{A}$ to deduce the time path for y from the time paths for $\tilde{y}$ and A.



- The basic intuition for why an increase in the growth rate of technology lowers steady state output per effective worker comes from the fact that at the old steady state, the break-even requirements are higher: with better technology, more investment needs to be used to keep capital per effective worker constant. Essentially, more investment is needed to make use of the faster growing technology.

- We can see, however that capital per worker and output per worker are both on higher growth trajectories even though it takes time for the economy to reach the higher growth part of g'.

- As before, we can use either math or intuition to determine the time paths of the variables K and Y from the time paths of k, y and L. Once again, I leave it up to you to show the following time paths for K and Y.



ln K
g′ + n
< g′ + n
g + n
t
t0
t1
ln Y
g′ + n
< g′ + n
g + n
t
t0
t1