Fall Semester '05-'06
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Lecture 11: Assessing the Solow Model with Technology

I. OVERVIEW

- In the last lecture we looked at the results of some comparative static exercises using the Solow model with technology. We showed that the impact of an increase in the savings rate is identical to the impact in the basic Solow model: there is a short-term increase in the growth rate of per-capita output but in the long run growth is unchanged. In other words, the increase in savings has a level effect, not a growth effect.
- We also looked at the impact of an increase in the growth rate of technology. The impact of an increase in the growth rate of technology has a growth effect: the steady state growth rate of output and capital per-capita is increased.
- In today's class, we will take a closer look at the steady state of the Solow model with technology. We then assess the predictions of the Solow model with technology and think about what avenues remain to be explored.

II. COMPARING STEADY STATES

- We can also do some comparative statics exercises using a little algebra instead of graphs. We know that at the steady state $\frac{\dot{\tilde{k}}}{\tilde{k}} = 0$. Therefore, using the capital accumulation equation $\dot{\tilde{k}} = s\tilde{y} - (n+g+\delta)\tilde{k}$ we can show

$$\begin{aligned} 0 &= s\tilde{y}^* - (n+g+\delta)\tilde{k}^* \\ (n+g+\delta)\tilde{k}^* &= s\tilde{y}^* \\ s\tilde{k}*^{\alpha} &= (n+g+\delta)\tilde{k}^* \\ \frac{s}{n+g+\delta} &= \tilde{k}*^{1-\alpha} \\ \Rightarrow \tilde{k}^* &= \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

- Using the fact that $\tilde{y} = \tilde{k}^{\alpha}$ we can then show that

$$\tilde{y}^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- Using the fact that $\tilde{k} = \frac{k}{A}$ and $\tilde{y} = \frac{y}{A}$ we can show that

$$k_t^* = A_t\left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \text{ and } y_t^* = A_t\left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- Note that in steady state, k and y are not constant, they are growing over time (hence the subscript t) because technology is growing.
- Finally, using the fact that $k = \frac{K}{L}$ and $y = \frac{Y}{L}$ we can show that

$$K_t^* = A_t L_t \left(\frac{s}{n+g+\delta} \right)^{\frac{1}{1-\alpha}} \text{ and } Y_t^* = A_t L_t \left(\frac{s}{n+g+\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- As is the case with k and y, K and Y are not constant in steady state either, they are growing over time both because technology is growing but also because the population is growing.

III. WHAT CAN THE SOLOW MODEL WITH TECHNOLOGY EXPLAIN?

- Now that we have an idea about the properties of the model, let's assess the intuitiveness of the predictions of the Solow model with technology.

1. Why are some countries rich and others poor?

- The Solow model with technology, like the basic Solow model, predicts that countries with high investment (savings) rates, low population growth rates and low rates of capital depreciation are likely to have more capital and output per worker.
- In addition, countries that have a high level of technology (A) or a high growth rate of technology (g) are also likely to be have more capital and output per worker in the steady state.
- The model also predicts that all else being equal, countries with a larger population are likely to be have more aggregate output which is the same conclusion reached by the basic model

2. Why do some countries grow faster than others?

- The Solow model with technology provides an explanation for differences in per capita economic growth across countries - in the long run all countries have a steady state output per capita growth rate that is equivalent to the growth rate of technology
- So countries with a high rate of technological progress are likely to grow faster.
- Even though the rate of population growth still has a positive impact upon the growth rate of the overall economy, the rate of technological progress also matters - the growth rate of Y is $n + g$.

3. How can economies exhibit sustained economic growth?

- In per capita terms, the Solow model with technology rectifies a major flaw of the basic model - it provides an explanation for why countries can grow in a sustained systematic manner for long periods of time: technological progress. Technology is the engine of economic growth.
- As in the basic model a country that increases its savings rate or lowers its population growth rate to increase the steady state level of output it can reach will see a positive spurt of growth in the short run.

- In addition a country that can increase its rate of technological progress will be able to move from a slow rate of growth to a high rate of economic growth even in the long run.

Conclusion

- The Solow model with technology preserves the basic framework of the simple Solow model but also provides a good intuitive framework for thinking about potential explanations for growth differentials and sustained economic growth.
- Even though the answers make good intuitive sense we need to verify if the model does a good job empirically - can it explain growth and level differentials across countries

IV. EMPIRICAL TESTS OF THE SOLOW MODEL WITH TECHNOLOGY

- Even though the predictions of the model make some intuitive sense, we would also like the model to be a good empirical model as well.
- A seminal paper by Gregory Mankiw, David Romer and Philippe Weil showed that the Solow model with technology can explain about 60% of the differences in economic growth across a sample of 98 countries: much of the explanation comes from differences in population growth rates and savings rates.
- They modify the model to include human capital (educational attainment) and show that the performance of the model is further enhanced. This modified model can explain about 78% of the differences in economic growth across countries.
- They model human capital as an input very similar to physical capital (K): a portion of savings is invested to accumulate human capital, some of which merely replaces depreciated human capital.
- However, we will limit our discussion of the specifics of their empirical findings because of econometric limitations and think of a different approach to test the empirical validity of the Solow model.
- For each country i in the sample, recall that (given data on savings rates, population growth rates etc.) we can write down an expression for steady state output per worker as

$$y_i^* = A_i \left[\frac{s_i}{n_i + \delta_i + g_i} \right]^{\frac{\alpha}{1-\alpha}}$$

- The level of technology in each country can be backed out from a Cobb-Douglas production function

$$\begin{aligned} Y_i &= K_i^\alpha (A_i L_i)^{1-\alpha} \\ \frac{Y_i}{K_i^\alpha L_i^{1-\alpha}} &= A_i^{1-\alpha} \\ \left[\frac{Y_i}{K_i^\alpha L_i^{1-\alpha}} \right]^{\frac{1}{1-\alpha}} &= A_i \end{aligned}$$

- The expression for y^*, may fail to pin down the scale of the economy i.e. we may not be able to calculate the exact $ value of per-capita GDP.

- Therefore we need to express steady state output per worker of the economy relative to a base country (the U.S. for example). Let $\hat{y}_i^*$ be the relative steady state output per worker of country i.

$$\hat{y}_i^* = \frac{A_i \left[\frac{s_i}{n_i+\delta_i+g_i}\right]^{\frac{\alpha}{1-\alpha}}}{A_{US} \left[\frac{s_{US}}{n_{US}+\delta_{US}+g_{US}}\right]^{\frac{\alpha}{1-\alpha}}}$$

- We can also calculate the actual relative income level for country i as well. For example the actual relative income in 1998 is given by $\hat{y}_i^{98}$ where we use the dollar value of real GDP per capita for each of the countries and for the U.S, i.e.

$$\hat{y}_i^{98} = \frac{y_i^{98}}{y_{US}^{98}}$$

- If the model's predictions are valid: a graph of $\hat{y}_i^*$ on $\hat{y}_i^{98}$ should show the observations clustered around a 45-degree line. See graphs shown in class and in the Jones textbook.

- Observations clustered around a 45 degree line confirm the validity of the Solow model only if all the economies are close to steady state.

- Economies that are away from steady state need not lie close to the 45 degree line because we are using the predicted steady state value of relative GDP.

- In drawing conclusions about the empirical validity of the Solow model from these charts, we should be aware that they depend on the following assumptions:

    1. $\delta + g$ are assumed to be the same across countries (technology transfers equalize g across countries?).
    2. An adjustment has been made for differences in human capital differences across countries, so that the actual methodology is a little more complicated than outlined above.
    3. We first assume A is the same across countries. (See Class Graph 1): the fit is not very good.[You can find this graph in Jones, Page 52] We then allow for differences in A across countries and find (see Class Graph 2) the fit is much better.[You can find this graph in Jones, Page 54]

V. OTHER TYPES OF TECHNOLOGY

- In addition to labor augmenting technology we can think of two other types of technology: capital augmenting technology and total factor productivity.

- **Capital Augmenting Technology**: Technology that works well with capital. Can be captured in a Cobb-Douglas production function as $Y = (CK)^\alpha L^{1-\alpha}$.

- **Total Factor Productivity**: Technology that works with all inputs (factors) to produce output. Can be captured in a Cobb- Douglas production function as $Y = BK^\alpha L^{1-\alpha}$.

- If we know how to work with the Solow model with labor-augmenting technology, we can easily handle the other types of technology as well.
- For example, we can rewrite the production function for capital augmenting technology so that it resembles a production function with labor augmenting technology as

$$\begin{aligned} Y &= (CK)^\alpha L^{1-\alpha} \\ &= K^\alpha \left(C^{\frac{\alpha}{1-\alpha}} L\right)^{1-\alpha} \end{aligned}$$

- Therefore we can see that this economy can be considered to be identical to a labor augmenting technology economy if $C^{\frac{\alpha}{1-\alpha}} = A$.
- In other words, if we are given an economy where capital-augmenting technology is growing at a rate g, then we can treat it as being equivalent to an economy where labor-augmenting technology is increasing at a rate $\frac{\alpha g}{1-\alpha}$. This is true because $C^{\frac{\alpha}{1-\alpha}} = A$ implies that

$$\frac{\dot{A}}{A} = \left(\frac{\alpha}{1-\alpha}\right)\frac{\dot{C}}{C} \equiv \left(\frac{\alpha}{1-\alpha}\right) g$$

- In the steady state of an economy where capital-augmenting technology is growing at a rate g, capital per worker and output per worker are increasing at a rate $\frac{\alpha g}{1-\alpha}$.
- We can also re-write a production function with TFP in terms of labor augmenting technology as

$$\begin{aligned} Y &= BK^\alpha L^{1-\alpha} \\ &= K^\alpha \left(B^{\frac{1}{1-\alpha}L} L\right)^{1-\alpha} \end{aligned}$$

- Therefore we can see that this economy can be considered to be identical to a labor augmenting technology economy if $B^{\frac{1}{1-\alpha}} = A$.
- In other words, if we are given an economy where TFP is growing at a rate g, then we can treat it as being equivalent to an economy where labor-augmenting technology is increasing at a rate $\frac{g}{1-\alpha}$. This is true because $B^{\frac{1}{1-\alpha}} = A$ implies that

$$\frac{\dot{A}}{A} = \left(\frac{1}{1-\alpha}\right)\frac{\dot{B}}{B} \equiv \left(\frac{1}{1-\alpha}\right) g$$

- Therefore in the steady state of an economy, where capital augmenting technology is growing at a rate g, capital per worker and output per worker are increasing at a rate $\frac{g}{1-\alpha}$.