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Lecture 10: Empirical Tests of the Expectations Hypothesis

I. OVERVIEW

- In the last lecture we studied some basic theory about the term structure of interest rates: the relationship between short and long-term interest rates in the economy. We discussed some basic features of bond pricing and used a theory known as the "expectations hypothesis", which stated that long-term interest rates are equal to the average of expected short-term interest rates over that term.

- The expectations hypothesis also provided an indication about how anticipation of future monetary policy decisions affect the economy. For example, when long-term bond yields are higher than short-term bond yields, according to the expectations hypothesis, short-term bond yields to rise over time.

- So if the Fed raises short-term interest rates today with an announcement that more interest rate hikes will follow in the near future (as they in fact did recently) then since short rates are expected to rise over time, then we would expect that long-term interest rates will rise as well. Higher long rates result in cutbacks of spending by firms and consumers slowing the economy.

- A theory that is as neat and intuitive as the expectations hypothesis is all too easy to accept at face value. However, it is important to verify if the theory does work as expected in reality. Today's paper, by Mankiw & Miron is one example of the myriad papers that seek to empirically test the validity of the expectations hypothesis.

II. WHY THE EXPECTATIONS HYPOTHESIS MAY NOT HOLD

- Consider the following relationship between yields on 1 year bonds and yields on 10 year bonds that the expectations hypothesis posits should hold

$$y_{10,t} = \frac{1}{10} E_t \left[y_{1,t} + y_{1,t+1} + \cdots + y_{1,t+9} \right]$$

- When we try to econometrically verify whether such a relationship between 1 year and 10 year bonds holds, we almost invariably reject the expectations hypothesis. In other words, even though the expectations hypothesis states that the return on long-term bonds should equal the expected return on a series of short-term bonds, we are unable to observe such a relationship in the real world.

- Why would this be the case? Well, one of the important reasons may be the fact that the expectations hypothesis is, as the name suggests, a story about how expected returns on a series of short bonds should equal the return on a long bond. In other words, we need to have data about what investors expected short rates to be at the time they were deciding between short and long-term bonds, which is difficult to come by.

- A second reason may be that borrowers and lenders are NOT indifferent between long-term bonds and a series of short-term bonds, as the expectations hypothesis assumes. For example, borrowers may prefer long-term loans (giving them greater security, more favorable payments etc.) while lenders may prefer short-term loans (allowing them more freedom to take advantage of new lending and business opportunities, have less chance of default etc.) as the famous economist John Hicks once postulated.

- In this case, the demand for bonds is likely to be higher at the short end of the term spectrum (since lenders/bond holders prefer short-terms) and the supply of bonds is likely to be higher at the long end of the term spectrum (since borrowers/bond issuers prefer long-terms). Therefore the price of short-term bonds is likely to be higher than the price of long-term bonds (since demand is greater) and since price is inversely related to yield, the yield on short-term bonds is likely to be lower than long-term bond yields. In other words, long-term bonds have to pay a higher yield to compensate for the lower demand.

- This results in the existence of something called the term-premium. Instead of $y_{10,t} = \frac{1}{10}E_t\left[y_{1,t} + y_{1,t+1} + \cdots + y_{1,t+9}\right]$, we would expect to find that

$$y_{10,t} = \theta + \frac{1}{10}E_t\left[y_{1,t} + y_{1,t+1} + \cdots + y_{1,t+9}\right]$$

III. THE MANKIW/MIRON PAPER

- The basic motivation for the Mankiw/Miron (MM) paper is the large body of research that documents the failure of the expectations hypothesis when tested using data. As MM point out, the rejection of the hypothesis goes back as far as 1938.

- In particular, MM are intrigued by the failure of the expectations hypothesis to hold for shorter-term treasuries of less than 1 year in duration. After all, the criticisms pointed out in the previous section, although valid for thinking about why the expectations hypothesis may not hold for 1 year and 10 year bonds, does not seem to be valid for thinking about why the expectations hypothesis does not hold for 3 month and 6 month bonds.

- Mankiw and Miron notice that most of these studies use data from the 1960s and 1970s. The research question they want to address is whether the rejections of the expectations hypothesis using data from the 1960s and 1970s are representative of other eras or whether there has been a change in the validity of the expectations hypothesis over time. If there has been a change, they would like to identify what factor may have been responsible for causing that change and thus shed some light on the enduring mystery about why the expectations hypothesis continues to be rejected even for studies that focus on short-term bonds.

- MM find that the expectations hypothesis, adjusted for a constant term-premium, seemed to hold in the period prior to the founding of the Federal Reserve. Following the founding of the Federal Reserve, they find that the expectations hypothesis does not hold, a fact which they attribute to the Federal Reserve's desire to smooth interest rates. The combination of interest rate smoothing and a slightly time varying risk-premium, they find, is sufficient to explain the failure of the expectations hypothesis to hold.

Setup

- The basic setup of the MM paper is very clear, and very simple. They work with two interest rates: a 3 month rate (the short rate) and a six month rate (the long rate). They use r to denote the short rate and R to denote the long rate. A quarter is considered to be 1 period.
- According to the expectations hypothesis, we would therefore expect that $R_t = \frac{1}{2}\left[r_t + E_t r_{t+1}\right]$ or allowing for the existence of a term premium that $R_t = \theta + \frac{1}{2}\left[r_t + E_t r_{t+1}\right]$.
- With a little bit of algebra this can be transformed into the equation

$$E_t r_{t+1} - r_t = -2\theta + 2(R_t - r_t)$$

- This equation says that when there is a **positive yield spread**, i.e. the yield on long bonds exceeds the yield on short bonds ($R_t > r_t$), then the yield on short bonds is expected to rise ($E_t r_{t+1} > r_t$). In other words when long-term interest rates are higher than short-term interest rates, short-term interest rates are expected to rise over time.
- Conversely, when there is a **negative yield spread**, i.e. the yield on long bonds is less than the yield on short bonds ($R_t < r_t$), then the yield on short bonds is expected to fall ($E_t r_{t+1} < r_t$). So when long-term interest rates are lower than short-term interest rates, short-term interest rates are expected to fall over time.
- If we assume that expectations are rational, then the expected future value will differ from the actual value only through information that is unavailable at time t, in other words $r_{t+1} = E_t r_{t+1} + v_{t+1}$ where v represents unforecastable factors that affect the short rate in the future.
- Using this relationship we can transform the equation $E_t r_{t+1} - r_t = -2\theta + 2(R_t - r_t)$ into a regression equation of the form $r_{t+1} - r_t = \alpha + \beta(R_t - r_t) + v_{t+1}$ where if the joint null hypothesis of rational expectations and the expectations hypothesis holds $\alpha = -2\theta$ and $\beta = 2$.

Data and Results

- To estimate this regression equation, MM use data from 1890 to 1979, they break this data up into 5 time periods: 1890-1914 (pre Federal Reserve), 1915-1933 (the end of the gold standard), 1934-1951 (the end of which signifies a change in the way the Federal Reserve operated), 1951-1958 (ending the sample at the point where most other studies begin), 1959-1979 (the period that is most studied by other economists.
- For each of the 5 time periods, MM run the regression equation given above, and the results are reported in Table I of the paper. The results show the following:
    1. The explanatory power of the yield spread between long and short rates for explaining future movements in short rates is very poor. The highest R-squared of 40% is for the period before the founding of the Fed. The highest R-squared in any other period is an abysmal 6%.
    2. The null hypothesis that $\beta = 2$ is rejected for all time periods: only in the pre-Fed period does it even approach the value of 2, with a value of 1.51. In several of the other periods, the coefficient is statistically indistinguishable from zero, indicating that the gap between short and long-term bond yields contained virtually zero information about the path of future interest rate movements.

- So how do MM explain the failure of the expectations hypothesis to hold? And how do they explain why the 1890-1914 period, the period before the founding of the Fed seems to be so different from the periods that followed?
- MM discuss two possibilities: the seasonal patterns exhibited by interest rates in the period before the founding of the Fed, or the prevalence of major financial panics in the early period. They find the controlling for the seasonality, and excluding the periods with financial panics does not make a difference in the performance of the expectations hypothesis in that early period.

Why Do We Always Seem to Reject the Expectations Hypothesis?

- Mankiw & Miron turn to two explanations for understanding the failure of these tests. The first is that the term-premium varies over time $\theta = \theta_t$. The second is that the variance of expected short-term interest rate movements is small. These two factors explain why we get regression coefficients close to zero when we run term-structure regressions.
- To understand what is going on, think back to your econometrics course. In a regression of the form $r_{t+1} - r_t = \alpha + \beta(R_t - r_t) + v_{t+1}$, the estimate of the slope parameter is

$$\beta = \frac{\text{Cov}[R_t - r_t, r_{t+1} - r_t]}{\text{Var}(R_t - r_t)}$$

- From the true model we know that $\frac{E_t r_{t+1} - r_t + 2\theta_t}{2} = (R_t - r_t)$ which gives us

$$\beta = \frac{\text{Cov}\left[\frac{E_t r_{t+1} - r_t + 2\theta_t}{2}, r_{t+1} - r_t\right]}{\text{Var}\left[\frac{E_t r_{t+1} - r_t + 2\theta_t}{2}\right]}$$

- Recall from QR 199 that given variables X, Y, Z and constants a, b, the following is true:
  1. Cov(aX, Y)=a*Cov(X, Y)
  2. Cov(X, X)=Var(X)
  3. Cov$(X, Y + Z)$)=Cov(X, Y)+ Cov(X, Z)
  4. Var(aX)=a^2Var(X)
  5. Var$(aX + bY)$=a^2Var(X)+b^2Var(Y)+2abCov(X, Y)
- Using this info, we can simplify the above down to

$$\beta = \frac{\text{Cov}\left[\frac{E_t r_{t+1} - r_t, r_{t+1} - r_t}{2}\right] + \text{Cov}\left[\theta_t, r_{t+1} - r_t\right]}{\frac{1}{4}\text{Var}\left[E_t r_{t+1} - r_t\right] + \text{Var}\left[\theta_t\right] + \text{Cov}\left[E_t r_{t+1} - r_t, \theta_t\right]}$$

- Under the assumption of rational expectations we have $r_{t+1} = E_t r_{t+1} + v_{t+1}$
- The slope coefficient is now

$$\beta = \frac{\text{Cov}\left[\frac{E_t r_{t+1} - r_t, E_t r_{t+1} - r_t + v_{t+1}}{2}\right] + \text{Cov}\left[\theta, E_t r_{t+1} - r_t + v_{t+1}\right]}{\frac{1}{4}\text{Var}\left[E_t r_{t+1} - r_t\right] + \text{Var}\left[\theta\right] + \text{Cov}\left[E_t r_{t+1} - r_t, \theta\right]}$$

- Since v_{t+1} is uncorrelated with time t variables this simplifies to

$$\beta = \frac{\text{Cov}\left[\frac{E_t r_{t+1} - r_t, E_t r_{t+1} - r_t}{2}\right] + \text{Cov}\left[\theta_t, E_t r_{t+1} - r_t\right]}{\frac{1}{4}\text{Var}\left[E_t r_{t+1} - r_t\right] + \text{Var}\left[\theta_t\right] + \text{Cov}\left[E_t r_{t+1} - r_t, \theta_t\right]}$$

- Using the shorthand notation $\Delta r_{t+1} = r_{t+1} - r_t$ and $E_t \Delta r_{t+1} = E_t r_{t+1} - r_t$ this becomes

$$\begin{aligned} \beta &= \frac{\frac{1}{2}\text{Var}\left[E_t \Delta r_{t+1}\right] + \text{Cov}\left[\theta_t, E_t \Delta r_{t+1}\right]}{\frac{1}{4}\text{Var}\left[E_t \Delta r_{t+1}\right] + \text{Var}\left[\theta_t\right] + \text{Cov}\left[E_t \Delta r_{t+1}, \theta_t\right]} \\ \Rightarrow \beta &= \frac{2\text{Var}\left[E_t \Delta r_{t+1}\right] + 4\text{Cov}\left[\theta_t, E_t \Delta r_{t+1}\right]}{\text{Var}\left[E_t \Delta r_{t+1}\right] + 4\text{Var}\left[\theta_t\right] + 4\text{Cov}\left[E_t \Delta r_{t+1}, \theta_t\right]} \end{aligned}$$

- Note that if the term premium is not time varying $\theta_t = \theta$, then the all covariance and variance terms involving θ become zero and we get $\beta = 2$, i.e. the null hypothesis holds. If the term premium varies, then the above equation shows that the regression coefficients are biased.
- The variance of the expected fluctuation in short-term interest rates affects the magnitude of this bias. As $\text{Var}[E_t \Delta r_{t+1}] \to \infty$

$$\beta \to 2$$

- On the other hand as $\text{Var}[E_t \Delta r_{t+1}] \to 0$

$$\beta \to 0$$

- In other words as the variance of $E_t \Delta r_{t+1}$ gets smaller, the regression coefficient gets biased away from 2 and closer to zero. On the other hand as the variance of $E_t \Delta r_{t+1}$ increases, the bias is gradually eliminated and the regression coefficient approaches 2.
- So even if the expectations hypothesis did in fact hold, when the variance of expected short-term interest rates is small AND we have a term-varying term-premium, a statistical test would conclude that the expectations hypothesis fails to hold.
- On the other hand if the expectations hypothesis did in fact hold and the variance of expected short-term interest rates is large, even with a term-varying term-premium, a statistical test would conclude that the expectations hypothesis holds.
- This is a very clever argument. MM have thus shown that it is plausible for the observed difference in results in the period prior to 1914 and the period following 1914, to be explained by the variability of expected short term interest rates.
- The results in Table III show that interest rates were in fact far more volatile in the period prior to the founding of the Fed and much smoother in the period following. Similarly the results in Figure III provide evidence that supports the idea that the estimated slope coefficient β exhibits the pattern described above, approaching a value of 2 as the variance of expected interest rate changes increases.
- Therefore tests of the expectations hypothesis post-1914 are much more likely to reject the expectations hypothesis than such tests done pre-1914, just as Mankiw & Miron demonstrated in Table I.

(Flawed) Intuition using the Notion of Predictability

- Mankiw & Miron try to give intuition to their piece by appealing to the notion of predictability. I think using the notion of predictability in this context can lead to very confusing intuition, and would ask that you steer clear.
- Let's review what we know: the bias is most pronounced when $\text{Var}[E_t \Delta r_{t+1}] \to 0$. The bias disappears when $\text{Var}[E_t \Delta r_{t+1}] \to \infty$. Note that $\text{Var}[E_t \Delta r_{t+1}] \to 0$ is equivalent to saying that $[E_t \Delta r_{t+1}] \to constant$
- But $[E_t \Delta r_{t+1}] \to constant$ can imply EITHER that interest rates are predictable or that they are unpredictable!
- Why? Well suppose the Fed was completely predictable in that it raises interest rates by 10 basis points each month. Then $[E_t \Delta r_{t+1}] = 0.10\%$ and $\text{Var}[E_t \Delta r_{t+1}] = 0$, hence the bias will be pronounced and $\beta = 0$
- In contrast, suppose the Fed was completely unpredictable so our best guess about what they would do in the future is the interest rate today, i.e. $E_t r_{t+1} = r_t$. Then $[E_t \Delta r_{t+1}] = 0\%$ and $\text{Var}[E_t \Delta r_{t+1}] = 0$, hence the bias will be pronounced and $\beta = 0$
- Therefore, its misleading to think about tests of the EH in terms of predictability or unpredictability. Its better to summarize the results as saying that when interest rates are expected to behave in a *volatile* manner ($\text{Var}[E_t \Delta r_{t+1}]$ is large) OR when there is no time varying term-premium, then tests of the expectations hypothesis are less likely to reject the EH when true.
- On the other hand, when interest rates are expected to move in a *smooth* manner ($\text{Var}[E_t \Delta r_{t+1}] \approx 0$) AND the term-premium is time varying, then tests of the expectations hypothesis are more likely to reject the EH even when true.
- In the period prior to the founding of the Fed, interest rates would fluctuate a lot - for example if a financial panic were to occur because shocks to money demand would drive short-term interest rates high. Since the short-term rates were much more volatile in this period, tests of the EH are most likely to confirm the EH when it is valid.
- On the other hand, the Fed tries to minimize such disruptions to avoid large fluctuations in interest rates - for example after Sept 11th, the Fed increased money supply substantially to meet the increased demand for money and to stop interest rates from rising. Since short-term rates are much less volatile in the face of active policy decisions by the Fed, tests of the EH are likely to reject the EH even if it is valid.
- The bottom line is that empirical tests of the EH can lead us to reject the EH even when true. Such rejections are likely when the Fed is smoothing interest rates and there is a time-varying term-premium.
- The complication we have to deal with is whether or not to adopt the EH as part of our toolkit. On the one hand, EH is an elegant, intuitive theory. Furthermore, we know that rejection of the EH can happen even if when the EH is true provided certain conditions hold. On the other hand, we have to be cautious about putting too much stock in a theory whose validity can't be conclusively proven empirically (in many ways the opposite of the theoretically suspect but empirically sound Phillips Curve).