

Mastery Grading

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MATHEMATICS DEPARTMENT

Background: Mastery Learning

- **Mastery Learning:** instructional strategy proposed by University of Chicago educational psychologist **Benjamin Bloom** in 1968. Its goal is to mimic the benefits of individualized tutoring, in a large class setting.
- Many versions of mastery learning have been implemented and studied since. They share two key features:
 - **Feedback, corrective and enrichment process:** formative assessment given at end of each unit; each student given either correctives or enrichment, as appropriate
 - **Instructional alignment:** specific learning goals and instruction that allows students to meet these goals
- In practice, implementing this can be difficult (e.g. if you cannot feasibly split the class into a “corrective” group and an “enrichment” group after each assessment).

Source: Guskey, T.R. (2007). Closing Achievement Gaps: Revisiting Benjamin S. Bloom's "Learning for Mastery." J. Advanced Academics, 19(1):8-31.

From Mastery Learning to Mastery Grading

Mastery Grading provides a way to realize some of the benefits of mastery learning by simply changing the grading system.

Each of us has implemented some form of mastery grading. While different, they share some basic features:

- Division of course material into “units” (by topic, or by week of class, etc.)
- Exams and quizzes test these areas specifically
- Students have multiple opportunities to demonstrate mastery of each unit, with opportunity to stop testing in an area once mastery is achieved
- Low or no penalty for taking repeated attempts to master something
- Grade based mostly on exams/quizzes

Alex's Mastery Grading Implementation

Megan Kerr and I taught 4 sections of Math 115 (Calculus I) together.

- Course material divided into 13 topics.
- We had 3 exams and a final. Each exam contained questions for each topic we had covered so far (new topics, plus old topics).
- On each exam, student work for each topic was scored either M (mastery), G (good), P(progressing) or X (could not be graded).
- Students also completed homework (twice weekly) and short quizzes (weekly). Students were essentially awarded full marks on these.
- A student's final grade was determined based on the number of M's and G's she had earned (using a student's best score for each topic).

Oscar's Mastery Grading Implementation

- Implemented in: Math 115, 116, 210, 215, and 303 (Summer 2017—Current semester)
- Weekly quizzes, each quiz divided into two topics; practice problems (with solutions) suggested for each lesson, but not graded (no homework)
- 2 exams plus self-scheduled final
 - Exam 1 contains the weekly quiz plus optional questions from topics covered to date
 - Exam 2 contains the weekly quiz plus optional questions from topics covered after Exam 1
 - Final contains comprehensive portion plus optional questions from all topics covered
- Current score on a topic is the largest of previous scores on that topic
- Weekly emails before quiz provide guidance on quiz content and standards used to grade quizzes

Oscar's Current Math 210 Course

Pre "Second Chance":

Math 210 Spring 2018 Grades

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100% \$ % .0 .00 123 Arial 10 B I A

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Banner ID		Topic 1 % Score	Topic 2 % Score	Quiz 1 % Score	Topic 3 % Score	Topic 4 % Score	Quiz 2 % Score	Topic 5 % Score	Topic 6 % Score	Quiz 3 % Score	Topic 7 % Score	Topic 8 % Score	Quiz 4 % Score	Topic 9 % Score	Topic 10 % Score	Exam 1 % Score
2	B20769256		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
3	B20810341		60	70	65	0	0	0	80	60	70	60	35	47.5	0	0	0
4	B20740130		80	40	60	80	100	90	85	85	85	80	50	65	100	100	100
5	B20857353		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
6	B20745908		100	100	100	90	100	95	90	90	90	100	70	85	95	100	97.5
7	B20737037		80	65	72.5	75	100	87.5	70	70	70	75	50	62.5	45	75	60
8	B20810003		100	100	100	100	100	100	80	90	85	70	90	80	100	100	100
9	B20765562		80	100	90	90	100	95	80	90	85	100	75	87.5	100	95	97.5
10	B20754673		80	90	85	95	100	97.5	70	100	85	90	50	70	100	85	92.5
11	B20746296		100	100	100	85	100	92.5	60	80	70	95	100	97.5	100	100	100
12	B20771899		90	100	95	100	100	100	80	95	87.5	90	70	80	100	0	50
13	B20821829		90	70	80	75	100	87.5	0	0	0	70	40	55	0	75	37.5
14	B20751983		100	100	100	90	100	95	80	100	90	0	0	0	100	75	87.5
15																	
16	Averages:		89.2	87.3	88.3	83.1	92.3	87.7	75.0	81.5	78.3	79.2	63.8	71.5	80.0	77.3	78.7
17																	
18	Pre-Exam 1 Averages:		89.2	87.3	88.3	83.1	92.3	87.7	75.0	81.5	78.3	79.2	63.8	71.5	80.0	77.3	78.7

Oscar's Current Math 210 Course

Pre "Second Chance":

Math 210 Spring 2018 Grades

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Banner ID		Topic 1 % Score	Topic 2 % Score	Quiz 1 % Score	Topic 3 % Score	Topic 4 % Score	Quiz 2 % Score	Topic 5 % Score	Topic 6 % Score	Quiz 3 % Score	Topic 7 % Score	Topic 8 % Score	Quiz 4 % Score	Topic 9 % Score	Topic 10 % Score	Exam 1 % Score
2	B20769256		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
3	B20810341		60	70	65	0	0	0	80	60	70	60	35	47.5	0	0	0
4	B20740130		80	40	60	80	100	90	85	85	85	80	50	65	100	100	100
5	B20857353		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
6	B20745908		100	100	100	90	100	95	90	90	90	100	70	85	95	100	97.5
7	B20737037		80	65	72.5	75	100	87.5	70	70	70	75	50	62.5	45	75	60
8	B20810003		100	100	100	100	100	100	80	90	85	70	90	80	100	100	100
9	B20765562		80	100	90	90	100	95	80	90	85	100	75	87.5	100	95	97.5
10	B20754673		80	90	85	95	100	97.5	70	100	85	90	50	70	100	85	92.5
11	B20746296		100	100	100	85	100	92.5	60	80	70	95	100	97.5	100	100	100
12	B20771899		90	100	95	100	100	100	80	95	87.5	90	70	80	100	0	50
13	B20821829		90	70	80	75	100	87.5	0	0	0	70	40	55	0	75	37.5
14	B20751983		100	100	100	90	100	95	80	100	90	0	0	0	100	75	87.5
15																	
16	Averages:		89.2	87.3	88.3	83.1	92.3	87.7	75.0	81.5	78.3	79.2	63.8	71.5	80.0	77.3	78.7
17																	
18	Pre-Exam 1 Averages:		89.2	87.3	88.3	83.1	92.3	87.7	75.0	81.5	78.3	79.2	63.8	71.5	80.0	77.3	78.7

Post "Second Chance":

Math 210 Spring 2018 Grades

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Banner ID		Topic 1 % Score	Topic 2 % Score	Quiz 1 % Score	Topic 3 % Score	Topic 4 % Score	Quiz 2 % Score	Topic 5 % Score	Topic 6 % Score	Quiz 3 % Score	Topic 7 % Score	Topic 8 % Score	Quiz 4 % Score	Topic 9 % Score	Topic 10 % Score	Exam 1 % Score
2	B20769256		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
3	B20810341		65	70	67.5	85	80	82.5	80	95	87.5	60	40	50	0	0	0
4	B20740130		80	95	87.5	80	100	90	85	85	85	80	90	85	100	100	100
5	B20857353		100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
6	B20745908		100	100	100	90	100	95	90	90	90	100	85	92.5	95	100	97.5
7	B20737037		80	70	75	75	100	87.5	85	85	85	80	90	85	45	75	60
8	B20810003		100	100	100	90	100	95	85	90	87.5	100	90	95	100	100	100
9	B20765562		100	100	100	95	100	97.5	100	100	100	100	75	87.5	100	95	97.5
10	B20754673		80	90	85	95	100	97.5	70	100	85	90	80	85	100	85	92.5
11	B20746296		100	100	100	85	100	92.5	100	100	100	95	100	97.5	100	100	100
12	B20771899		95	100	97.5	100	100	100	100	95	97.5	90	100	95	100	0	50
13	B20821829		90	70	80	75	100	87.5	90	95	92.5	95	40	67.5	0	75	37.5
14	B20751983		100	100	100	90	100	95	80	100	90	95	15	55	100	75	87.5
15																	
16	Averages:		91.5	91.9	91.7	89.2	98.5	93.8	89.6	95.0	92.3	91.2	77.3	84.2	80.0	77.3	78.7
17																	
18	Pre-Exam 1 Averages:		89.2	87.3	88.3	83.1	92.3	87.7	75.0	81.5	78.3	79.2	63.8	71.5	80.0	77.3	78.7
19	Change after Exam 1:		2.3	4.6	3.5	6.2	6.2	6.2	14.6	13.5	14.0	11.9	13.5	12.7	0.0	0.0	0.0

Observed Benefits of Mastery Grading

- Students persist. Few drop. Many pass.
- Students are more willing to sustain engagement with difficult material, rather than just giving up.
- Mastery grading feedback helps students focus more on improving learning in specific areas, rather than just accumulating points.
- It fosters a more “individualized” assessment of each student's progress. Within a single class, different students can learn different topics at different paces.
- Mastery grading encourages improvement and growth.
- The novelty of the grading marks can refocus student attention away from grades (a little).

Possible Drawbacks and Pitfalls of Mastery Grading

- Students might resist unfamiliar grading scheme; you need to be ready to “sell” it.
- In some implementations, students could decide to “check out” once they have mastered enough topics.
- Dividing course into “units” may seem a bit forced; it takes some care.
- Instructor may need to rethink topic ordering, especially in disciplines which are highly cumulative.
- Instructor will need to write A LOT of exam/quiz questions for some topics.
- Depending on the implementation, you may decide to offer students extra time on later exams. This can become a scheduling issue.

Calculus and Mastery Learning

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Department of Mathematics
March 16, 2018

Reasons for reconfiguring calculus courses

- Students with varying mathematics backgrounds
- Students from underresourced high schools
- Barrier for students in sciences
- Difficulty applying mathematics correctly in science courses

Most pernicious mathematics exercises: word problems

- Rate, time and distance
- Combined rates (work)
- Optimization (Calculus 1)
- Energy (Calculus 2)
- Hydrostatic Pressure (Calculus 2)

Examples

- Charles Bu flies to Shanghai to Boston in 12 hours. The distance covered is 11,725 kilometers. What is the rate that the plane is flying?
- Professor Trenk can grade 100 exams in 10 minutes, and Professor Kerr can grade 100 exams in 12 minutes. Working together, how quickly can they grade 100 exams?
- Suppose that a farmer has 100 feet of fencing and wants to build an enclosure along a river. What dimensions give the maximum area?
- A chain is attached to a 20-lb bucket, which is resting at the bottom of a 40-foot well. The chain has a density of 10 pounds per foot. How much energy (work) does it take to pull the bucket out.

Prerequisites for Calculus 115

- Manipulating fractions
- Manipulating exponents
- Simplifying algebraic expressions
- Solving systems of equations
- Finding roots of polynomials
- Exponential function
- Logarithm function
- Trigonometric functions
- Inverse trigonometric functions
- The unit circle
- Trigonometric identities
- Equations for parabolas, circles, ellipses and hyperbolas
- Properties of spheres, cones, cylinders
- Graphs of all the functions above

The unfortunate result

- Students drop out during the first week.
- Students withdraw after the drop deadline.
- Students move away from science.
- Students move away from other quantitative-based majors.
- A disproportionately small number of underrepresented minorities inhabit the Science Center.

Mastery Learning

- The course is divided into 15 “Skills”.
- The first exam covers the first 5 of these skills.
- The second exam covers the first 10 of these skills.
- The third exam covers all 15 skills.
- The final covers all 15 skills.
- A skill, once mastered, never has to be redone.
- An unmastered skill can be attempted at the next exam.
- Students can elect not to take the final without penalty.

Midterm Examination 1: Mathematics 115z

Name: _____

This exam is to be completed in 70 minutes without the use of calculators or your notes. Please be reminded that the Honor Code requires that you submit work that is entirely your own. Please write on the pages provided. Write your name on each sheet.

1. Skill 1

- (a) Use interval notation to find the domains of functions $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{1}{\sqrt{x}}$.
- (b) Let $f(x) = x^2 + x$. Let $g(x)$ be the function that shifts $f(x)$ one unit to the right and then two units up. Write a formula for $g(x)$.

2. Skill 2

- (a) Let $f(x) = 3x^2 + \sin x$ and $g(x) = e^{4x}$. Compute both $f(g(x))$ and $g(f(x))$.
- (b) Let $f(x) = e^{3x} + 4$. By switching the positions of x and y , find the inverse function $f^{-1}(x)$. Note: We usually call the inverse function $g(x)$, so use either notation.

3. Skill 3

Consider the piecewise-defined function given by

$$f(x) = \begin{cases} 3x^2 & \text{if } 0 \leq x \leq 1, \\ 4 - x & \text{if } 1 < x \leq 4, \\ \frac{1}{x-4} & \text{if } x > 4. \end{cases}$$

- (a) Find the domain of f in interval notation.
- (b) Write down the equation of all asymptotes (both horizontal and vertical).
- (c) Provide a graph of this f .
- (d) By computing $\lim_{x \rightarrow 1^+} f(x)$, $\lim_{x \rightarrow 1^-} f(x)$ and $f(1)$, decide whether f has a discontinuity at $x = 1$. If it is discontinuous at $x = 1$, decide whether it is removable or unremovable.
- (e) By computing $\lim_{x \rightarrow 4^+} f(x)$, $\lim_{x \rightarrow 4^-} f(x)$ and $f(4)$, decide whether f has a discontinuity at $x = 4$. If it is discontinuous at $x = 4$, decide whether it is removable or unremovable.

4. Skill 4

Let $f(x) = \frac{3x^2+1}{x^2-2x}$

- (a) By factoring the denominator, find the domain of the function in interval notation.
- (b) Find the equations all asymptotes, both horizontal and vertical.
- (c) For each of the two points not in the domain, calculate the limit of the function from both the left and right sides. [So you will calculate four limits.]
- (d) Draw the pieces of the function described by all the information in (a)-(d). You do not need to draw the whole function.

5. Skill 5

- (a) Suppose that a radioactive product has half-life given by 100 years. Suppose that we begin with 400 grams of this substance. How long will it take for it to decay to 25 grams? Please leave your answer in unsimplified form.
- (b) A tank of water contains no pollutants at $t = 0$. At $t = 5$, a large amount of pollutants is dumped into the tank and mixed. At $t = 10$, a valve is opened at the bottom of the tank that drains the mixed liquid, while a valve at the top is opened, replenishing the tank with fresh water so that the volume stays the same. At $t = 15$, both valves stop functioning, so no water movement takes place. At $t = 20$, both valves start up again. Draw a graph of time versus amount of pollutants over the domain $[0, \infty)$.

I. Basic function properties, domain, shifting and reflections.

- (a) Find the domains of $f(x) = \frac{1}{(x+2)^2(x-5)}$ and $g(x) = \frac{1}{\sqrt{3-x}}$.
- (b) Let $f(x) = e^{2x} - 4x^2$. Find the function g that is obtain by shifting f to the left 3 units, then shifting down by 2 units, then reflecting across the x -axis.

II. Compositions and inverse functions

- (a) Let $f(x) = x + \ln x$ and $g(x) = x^3 + 4$. Compute $f(f(x))$ and $f(g(x))$.
- (b) Let $f(x) = 2e^{x^3-5}$. Find the inverse $g(x)$ of $f(x)$. Then show that $f(g(x)) = x$ and $g(f(x)) = x$.

III. Piecewise defined functions

- (a) Determine the domain of the function f and sketch the function on its domain.

$$f(x) = \begin{cases} 1 & \text{if } -3 \leq x \leq 1, \\ \ln x & \text{if } 1 < x \leq 3, \\ x+1 & \text{if } x > 3. \end{cases}$$

- (b) Write the equation of all asymptotes, and find all points of discontinuity. Determine whether the discontinuities are removable or unremovable.

IV. Asymptotes

- (a) Consider the function $f(x) = \frac{1}{(x-1)^2(x-3)}$. Find the two vertical asymptotes and compute the left and right limits of f for both of the asymptotes.
- (b) By computing $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$, find the horizontal asymptote. Use all the information to sketch the graph of f .

V. Word problems involving exponentials and logarithms, general sketching

- (a) A radioactive material decays to one-third its original amount every 14 years. If there are 40 grams of the material at $t = 2$ years, find the original amount.
- (b) From January 1, 2016, the immigration office in Canada has been receiving incrementally more phone calls from Americans determined to immigrate to Canada as Trump is elected president. After Hillary is elected on November 8, 2016, all phone calls to Canada stop. Give a possible graph of time versus phone calls from January 1 to December 31.

VI. Computing derivatives using the Method I/II definitions

- (a) Compute $f'(a)$ if $f(x) = 4x^2 + 1$ using Method I.
- (b) Compute $f'(x)$ if $f(x) = \sqrt{3-x}$ using Method II.

VII. Derivatives of polynomials, exponential and logarithmic functions, using chain rule

- (a) Find all the points x for which the tangent line of $f(x) = \frac{1}{3}x^3 - 2x^2 + 4x$ has slope 4.
- (b) Let $f(x) = e^{2x} + \ln(4x+3) + 2\sqrt{x+1}$. Compute $f'(0)$ and write the answer in its simplest form. Write the tangent line at $a = 0$ in point slope form.

VIII. Critical points, concavity, curve sketching

- (a) Find all roots, critical points and inflection points of $f(x) = x^3 - 9x$.
- (b) Use number lines to decide where the function is above/below the x -axis, increasing/decreasing, concave up/down. Sketch the curve.

IX. Word problems involving optimization

- (a) Suppose that A people are willing to buy a product at the price B . Suppose that, for every dollar increase in price, the seller loses C customers. Write down the equation of the revenue made when x dollars are added to the original cost A . Find the value of x that would generate the most revenue.
- (b) Suppose that a cylinder must have a volume of 200 cubic inches. Find the radius and the height of the cylinder with the least surface area.

X. The product rule and quotient rule

- (a) Compute the derivatives of the following: (i) $f(x) = x^3 \ln x$, (ii) $f(x) = \frac{(x^2+3x)^5}{e^{2x}+4 \ln x}$.
- (b) Let $f(x) = xe^x$. Compute $f'(x)$ and $f''(x)$ and find all roots, critical points and inflection points. Draw a number line for each and determine positive/negative, increasing/decreasing and concave up/down. Sketch the function.

Other grading schemes

- Last year, each skill was scored M, G, P, X.
- Numerically M=20, G=15, P=5, X=0.
- Class grade: 65% exam, 30% homework, 5% quizzes
- This year, each skill is scored out of 10, with the usual conversion A=9, B=8, C=7, D=6.

Mathematics 116: Spring 2017

P		P	M	M	M	X	M	M	M	G	M	P	M	M	P	M	P	G	M	G	P	M	M	M	M
X	M	M	M	M	M	M	M	M	M	P	M	M	M	G	G	G	M	G	M	P	M	M	M	G	G
G	M	M	M	M	P	M	M	G	M	M	G	M	P	M	M	G	M	M	M	G	M	M	M	M	M
P	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
P	M	M	M	M	M	M	M	G	P	P	P	M	G	G	P	G	M	M	G	G	P	P	G	G	G
P	M	G	M	M	X	M	G	M	G	P	P	G	M	G	M	G	X	P	G	P	G	P	G	G	G
P	M	M	M	M	M	G	G	M	M	P	M	M	M	M	M	M	P	M	M	P	M	M	M	M	M
P	M	P	P	M	P	M	M	P	P	P	P	M	P	M	P	X	P	P	M	X	X	G	P	M	M
P	M	M	P	M	X	G	M	M	X	X	P	M	P	P	P	G	G	G	X	P	X	P	M	P	P
P	M	G	G	G	P	M	M	G	P	X	G	M	G	P	M	G	P	M	P	G	P	M	G	X	X
X	G	G	P	M	X	G	M	P	M	X	P	G	G	M	P	P	G	G	X	M	X	P	G	P	P
P	M	G	M	M	M	M	M	P	M	G	P	G	M	G	M	M	M	P	G	M	P	G	G	M	P
X	M	G	G	G	G	M	M	P	G	P	P	M	P	M	P	G	G	G	G	P	P	M	M	G	G
X	M	M	P	M	X	X	M	M	G	X	M	M	M	G	G	X	X	P	G	P	X	P	M	P	P
X	G	G	P	M	P	G	G	P	P	X	X	M	G	G	P	M	X	X	P	P	M	X	P	M	P
0	12	8	8	13	4	10	13	7	4	5	4	13	5	7	6	4	4	7	4	5	5	7	9	5	
5	0	0	0	0	5	1	0	0	1	5	1	0	0	0	0	2	3	0	2	1	5	0	0	1	
1	2	6	2	2	2	4	2	3	4	1	3	2	6	5	3	6	5	4	2	5	3	2	2	5	4
9	1	1	5	0	4	0	0	5	6	4	7	0	4	3	6	3	3	4	4	6	3	6	1	5	
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
20	92	85	72	97	43	87	97	70	57	45	53	97	70	77	65	62	57	73	58	58	48	67	87	62	
52	95.2	91	83.2	98.2	65.8	92.2	98.2	82	74.2	67	71.8	98.2	82	86.2	79	77.2	74.2	83.8	74.8	74.8	68.8	80.2	92.2	77.2	
F	A	A-	B	A	D	A-	A	B-	C	D	C-	A	B-	B	C+	C+	C	B	C	C	D	B-	A-	C+	

Mathematics 115: Fall 2017

Mid1	Mid2	Mid3	Mid4	Mid5	Mid6	Mid7	Mid8	Mid9	Mid10	Mid11	Mid12	Mid13	Mid14	Mid15		
10	6	8	9	7	3	10	8	10	10	5	0	3	0	7	64	C-
10	8	6	7	10	6	9	10	10	10	7	8	9	8	7	83.33333333	B
7	7	10	10	7	10	10	10	8	8	6	6	10	10	10	86	B+
8	4	10	7	9	10	10	10	10	10	6	10	9.5	10	10	89	A-
10	9	10	8	9	10	9	10	10	4	7	3	10	0	5	76	C+
10	8	8	9	9	10	10	5	3	10	10	2	6	9	10	79.33333333	B-
10	10	10	10	10	10	10	10	10	10	10	10	9	10	7	97.33333333	A
8	7	8	8	9	8	10	8	10	5	4	4	10	10	7	77.33333333	B-
9.5	10	8	9	10	10	10	10	10	10	8	9	10	10	10	95.66666667	A
10	9	10	10	8	9	10	10	9	6	9	10	10	10	7	91.33333333	A-
9.25	7.80	8.80	8.70	8.80	8.60	9.80	9.10	9.00	8.30	7.20	6.20	8.65	7.70	8.00	8.393333333	