

## The Burden of Proof

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Two number theorists are taking on fellow mathematician Shinichi Mochizuki of Kyoto University and his world-famous proof, saying it is not a proof at all.

The nearly seven-year old proof in question deals with a mathematical statement called the “abc conjecture,” and spans a series of four papers to controversially claim the conjecture true. Now, recent Fields Medalist Peter Scholze of the University of Bonn and Jakob Stix of Goethe University Frankfurt declare that Mochizuki’s proof has a hole.

The conjecture has to do with the most familiar operations on numbers: addition and multiplication. While mathematicians admittedly know a lot about addition and multiplication, number theorists are keen to know more about how the two operations are connected, as it can give them a deeper understanding of the hidden structures of math.

If true, the conjecture ends up saying something about how multiplication (via numbers’ prime factors) and addition are related. The name “abc” itself comes from the addition piece in the conjecture: the equation  $a + b = c$ . The general idea is that this relationship produces a number  $c$  which is almost always smaller than all of the distinct prime factors of  $a$ ,  $b$ , and  $c$  multiplied together. For example, take  $4+3 = 7$ . From 4, 3, and 7, we find distinct prime factors 2, 3, and 7 to see that  $2 \times 3 \times 7 = 42$ . As predicted by the conjecture, 42 is larger than 7.

Since the conjecture’s proposal in 1985, only two serious proof attempts have been put forward. The math community quickly found an error in the first of the two. The second is Mochizuki’s proof.

Given both the simplicity of statement and the difficulty in proving it, the conjecture has been of particular interest to number theorists. Thus, when self-proclaimed “[inter-universal geometer](#)” Mochizuki posted his purported proof of the statement, experts in the field were eager to read through it, despite it being over 500 pages in length.

However, Mochizuki’s proof has brought little clarity to the problem. Ivan Fesenko of the University of Nottingham said of the mathematicians who have read deeply into the proof, [only 12 to 18 \(including himself\) believe it is correct](#). Which is far from persuading the entire number theory community.

The trouble is, none of those who believe the proof is correct have been able to fully convince others in the field that it is logically sound. Those reluctant to side with Mochizuki, cannot tell if the confusion is due to a failure in exposition or in mathematical reasoning. In particular, Scholze and Stix, who now come forward in denouncing the proof, claim that they are able to follow the proof until the third paper. It is at this point where they believe there is a hole and, as a result, insist “[there is no proof.](#)” However, throughout his career, Mochizuki has proved other important results and was

an invited speaker at the International Congress of Mathematicians, an ultimate mark of accomplishment in the field. Given Mochizuki’s reputation, most mathematicians are reluctant to go on the record about the proof. Thus, the community is at an impasse: the proof is neither accepted nor seen as entirely invalid.

“I tried to read some of [Mochizuki’s papers] and then, at some stage, I gave up. I don’t understand what he’s doing,” said Gerd Faltings to [Scientific American](#). “It’s not enough if you have a good idea: you also have to be able to explain it to others.”

Like Scholze, Faltings is a recipient of the Fields Medal, the highest honor awarded in mathematics. Faltings is also Mochizuki’s former Ph.D. advisor.

Scholze and Stix even traveled to Japan early in 2018 to discuss their objections with Mochizuki. In a [report](#) following the weeklong visit, Mochizuki wrote that when recounting to his colleagues the disagreements Scholze and Stix had with the proof, he was “met with a remarkably unanimous response of utter astonishment and even disbelief (at times accompanied by bouts of laughter!) that such manifestly erroneous misunderstandings could have occurred.”

Since their meeting with Mochizuki, Scholze and Stix wrote their own report titled [“Why abc is still a conjecture”](#) this past September.

“[I]n our opinion, the suggested proof has a problem, a problem so severe that in our opinion small modifications will not rescue the proof strategy,” said Scholze and Stix in their report.

Mochizuki has not budged. He asserts that the “negative position” of Scholze and Stix “does not imply the existence of any flaws whatsoever” in his work. And although Scholze and Stix have left the ball in Mochizuki’s court, Scholze notes that the fate of the conjecture does not necessarily lie solely with Mochizuki:

[“I think the abc conjecture is still open,”](#) said Scholze. “Anybody has a chance of proving it.”