Study Packet for the **Quantitative Reasoning Assessment**



Study Packet for the Quantitative Reasoning Assessment

Dear Incoming Wellesley Student:

Hello! We have prepared this study packet to help you prepare for the Quantitative Reasoning (QR) Assessment that you will take soon after arriving on campus.

What is the QR Assessment for?

The QR Assessment tests your quantitative skills, including your ability to read and understand information presented in formulas, tables, and graphs; to interpret information and draw appropriate inferences; and to solve real world problems that deal with numbers or data. The mathematical skills you will apply on this test span arithmetic, algebra, graph reading, geometry, and linear and exponential modeling. Logical and statistical skills are core "QR skills." You need to demonstrate adequate basic QR skills before enrolling in a wide array of first-year courses at the College. Additionally, strong QR skills are needed to ensure that you can explore any academic major, pursue any career, and address the wide array of quantitative problems that arise in everyday life.

What happens if I do not do well on the QR Assessment?

If you do not pass the QR Assessment, you will need to enroll in "Introduction to Quantitative Reasoning" in your first year at the College. After successfully completing this course, you should have the quantitative skills that are a prerequisite for any introductory level course that emphasizes quantitative analyses - from science courses to economic courses.

What is included in this study packet?

This study packet includes a set of 24 review problems (starting on page 3) and two QR Assessments actually given to students in past years (starting on page 27). Short answers are provided for each problem. In addition, complete, worked-out solutions are provided for all but the last actual assessment. We suggest you work through these problems without using a calculator, as calculators are NOT allowed when taking the 90-minute QR Assessment. If there are particular types of problems that you find challenging, we recommend reviewing notes or books on these topics.

If I have questions, where should I turn?

For more information about Wellesley's two-part QR requirement or about the QR Assessment itself, visit the QR Web site at www.Wellesley.edu/QR/. The Web site lists recommended readings, etc. You may also contact Corri Taylor, Director of the QR Program, by e-mail at *ctaylor1@wellesley.edu* or by phone at 781.283.2152.

Review Problems

- partygoer leave behind?

- years 2001, 2002, and 2003.
- decrease between 1990 and 1994?
- married Americans in 1970? Simplify your answer.
- proportion.)

1. Officials estimate that 320,000 Boston-area partygoers attended last year's Independence Day celebration on the banks of the Charles River. They also estimate that the partygoers left behind 40 tons of garbage. Given that a ton equals 2,000 pounds, how many pounds of garbage did the average

2. Subway tokens cost 85 cents. How many can you buy with \$20?

3. A student's grade depends on her score on four equally-weighted exams. Her average on the first three exams is 92. What must she score on the fourth exam in order to guarantee a final average of at least 90?

4. In 2000, a person places \$1,000 in an investment that earned 10% annual interest, compounded annually. Calculate the value of the investment for the

5. According to The New York Times, scientists studying the atmosphere have recently detected a decrease in the level of methyl chloroform, a man-made industrial solvent that is harmful to the ozone layer. In 1990, the level of methyl chloroform was 150 parts per trillion (150 ppt), but by 1994 the level had fallen to 120 ppt. By what percentage did the level of methyl chloroform

6. In 1990, according to Census data, one in four Americans over 18 years of age had never married, as compared to one in six in 1970. What is the ratio of the fraction of never married Americans in 1990 to the fraction of never

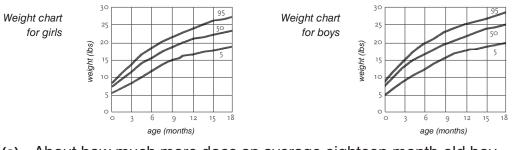
7. One year ago, a person invested \$6,000 in a certain stock. Today, the value of the investment has risen to \$7,200. If, instead, the person had invested \$15,000 one year ago instead of \$6,000, what would the investment's value be today? (Assume that the investment would increase by the same

8. Evaluate the following expressions given that v = -2 and w = 3

(b) $v^2 + w^2$ (a) 3(v - 2w)

9. Figure 1 gives weight charts for baby boys and girls from birth to 18 months. Each chart gives weights for three different sizes of babies: 5th percentile (small babies), 50th percentile (average babies), and 95th percentile (large babies). For example, according to the chart, the 50th percentile weight for a six-month-old girl is about 15 pounds, while the 95th percentile weight for a six-month-old girl is about 18 pounds.

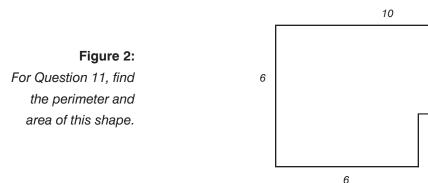
Figure 1: Weight charts for baby boys and girls, age birth to 18 months

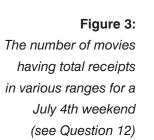


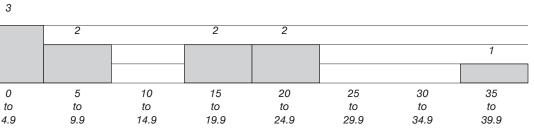
- (a) About how much more does an average eighteen-month-old boy weigh than an average eighteen-month-old girl?
- (b) Consider two 6-month-old boys, one in the 5th percentile and one in the 95th percentile. About how old will the smaller boy be when he weighs as much as the larger boy does now? (You should assume that the smaller boy remains in the 5th percentile as he grows.)

4

- 10. In 2003, there were 80 turtles living in a wetland. That year, the population began to grow by 12 turtles per year. Find a formula for P, the number of turtles in terms of *t*, the number of years since 2003.
- **11.** Find (a) the perimeter and (b) the area of the shape in Figure 2.







- at agency *B* is given by $C_B = 12 + 0.40n$.
 - less, A or B?
 - less expensive?
- brought in by the airline for this flight.

12. Figure 3 shows the number of movies with weekend receipts in different dollar ranges for a July 4th holiday weekend. For example, according to the chart, two movies earned at least \$15 million but less than \$20 million. According to Figure 3, how many movies earned more than \$10 million?

weekend receipts (\$ millions)

13. Suppose you need to rent a car for one day and that you compare that cost at two different agencies. The cost (in \$) at agency A is given by $C_{4} = 30 + 0.22n$, where n is the number of miles you drive. Similarly, the cost

(a) If you drive only a few miles (say, less than 20 miles), which agency costs

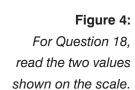
(b) How far would you need to drive in order for the other agency to become

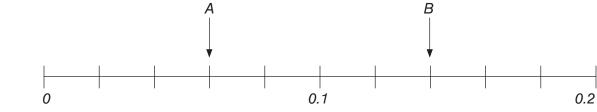
14. According to the Cable News Network, the number of injured in-line skaters (or "roller-bladers") was 184% larger this year than it was last year. Did the number of injured skaters almost double, almost triple, or more than triple?

15. For a certain flight out of Chicago, let P, be the price of a first-class seat and P_c be the price of a coach-class seat. Furthermore, let N_f be the number of first-class seats and N_c be the number of coach-class seats available on the flight. Assuming that every available seat is sold, write an expression in terms of these constants that gives the value of R, the total amount of money

16. Solve the equation
$$\frac{Z_1}{Z_2} = \frac{K_1}{K_2} \quad \text{for } K_2.$$

- 17. There are 0.6 grams of powder in a dish. One-fifth of the powder spills out of the dish. How many grams of powder are left in the dish?
- **18.** Read the values at the two pointers shown in Figure 4.





- **19.** A six-foot tall man is walking home. His shadow on the ground is eight feet (ft) long. At the same time, a tree next to him casts a shadow that is 28 ft long. How tall is the tree?
- **20.** Graph the equation 5s + 15 t = 0 on the set of axes provided in Figure 5. Label the s- and t- intercepts.

For Question 20, graph the equation 5s + 15 - t = 0 on this set of axes.

S Figure 5:

21. Express the following values in simplified form, in scientific notation.

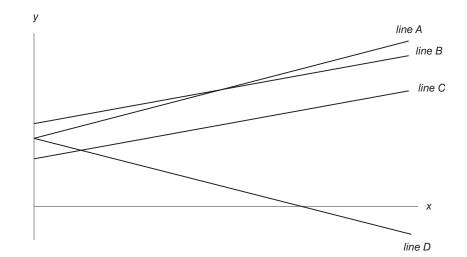
(a) $(2 \times 10^{-4}) (3 \times 10^{5})$

- descriptions that follow.
 - (i) P = 1000 50t
 - (iii) P = 1000 + 70t
 - (a) This population decreases by 5% each year.

23. Match the following equations to the graphs shown in Figure 6.

- (a) y = 6 0.8x
- (c) y = 4 + 0.5x

Figure 6: Match the equations given in Question 23 on these graphs.



(b)
$$(3 \times 10^{-4})(8 \times 10^{5})$$

 4×10^{-7}

22. The equations below describe several different animal populations over a period of time. In the equations, *P* stands for the size of the population and *t* stands for the year. Match the appropriate equation or equations to the verbal

(ii) $P = 8000(0.95)^t$

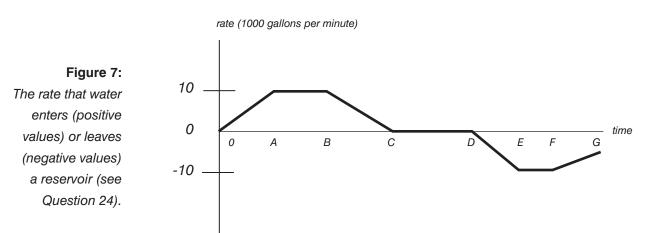
(iv) $P = 5000 + 2000 \sin(2\pi t)$

(b) This population increases by the same number of animals each year.

(c) This population rises and falls over the course of the year.

(d) In year t = 0, these populations are at the same level.

(b) y = 6 + 0.9x(d) y = 8 + 0.5x 24. Figure 7 gives the rate (in thousands of gallons per minute) that water is entering or leaving a reservoir over a certain period of time. A positive rate indicates that water is entering the reservoir and a negative rate indicates that water is leaving the reservoir. State all time intervals on which the volume of water in the reservoir is increasing.



STOP!!!

- usually a much more valuable learning experience.

• Don't read these solutions until you have tried the problems on your own!

• Short answers are provided on page 11. Check these first.

• Worked-out solutions have also been provided, starting on page 13.

• *Tip.* Work the problems and then check the short answers. If you miss a question, try to figure out how to solve it on your own before you read the complete, worked-out solution. You may feel the urge to read the worked-out solutions right away, but discovering the answer to a problem on your own is

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Short Answers to Review Problems

1. 1/4 pound	
2. 23 tokens	
3. 84	
4. \$1,100 in 2001, \$1,210 in 200)2, \$
5. 20%	
6. 3/2 or 1.5	
7. \$18,000	
8. (a) -24	(b)
9. (a) about 2 lbs more	(b)
10. $P = 80 + 12t$	
11. (a) Perimeter is 32 units.	(b)
12. five movies	
13. (a) agency <i>B</i>	(b)
14. almost triple	
$15. R = N_f P_f + N_c P_c$	
16. $K_2 = K_1 Z_2 / Z_1$	
17. 0.48 grams	

002, \$1,331 in 2003

(b) 13

(b) about 18 months old

(b) Area is 52 squared units.

(b) more than 100 miles

19.21 feet tall

20. The s-intercept is -3 and the *t* intercept is 15. See Figure 7 on page 22.

21.	(a) 6.0 × 10	1	(b) 6.0 × 10 ⁸	
22.	(a) ii	(b) iii	(c) iv	(d) i and iii
23.	(a) line D	(b) line A	(c) line C	(d) line <i>B</i>

24. from time 0 to time C

Worked Solutions to Review Problems

pounds (lbs). That makes

We can find the average amount of garbage that each person left behind by dividing the 80,000 pounds of garbage among the 320,000 partygoers:

Thus, the average partygoer left behind one quarter of a pound of garbage.

- to be exact).

Exam

То

Now, if the student wants her final average for all 4 exams to be at least 90 points, then her point total for all 4 exams must at least $4 \times 90 = 360$. Since she already has 276 points, she only needs 360 - 276 = 84 points more. Thus, she must score at least an 84.

1. The partygoers left behind 40 tons of garbage, each ton weighing 2000

40 tons \times 2,000 lbs. = 80,000 lbs. ton

> 80,000 lbs. = 1/4 lb. per person. 320,000 people

2. Since $$20 \div 0.85 = 23.5$ (to one decimal of accuracy), this means you can buy 23 tokens with \$20 and expect some change. Another way to work this problem is to think of buying tokens in sets of 10. One set of 10 tokens costs $10 \times \$0.85 = \8.50 . This means that two sets of 10 tokens would $cost 2 \times \$8.50 = \17 . So if you buy 20 tokens for \$17, you still have \$3 left. This is only enough to buy 3 more tokens, because $3 \times 0.85 = 2.55$. In conclusion, \$20 will buy 23 tokens and leave you with some change (\$0.45,

3. Think about how you calculate an exam average: the point total of all of your exams is divided by the number of exams. Since this student's average for her first 3 exams is 92 points, this means that her point total for these 3 exams is $3 \times 92 = 276$ points. If you're not sure about this step, notice that

n average	=	total score no. exams
otal score	=	exam average × no. exams
	=	92 × 3 = 276.

4. Table 1 shows the bank balance for 2000, 2001, 2002, and 2003.

Table 1:	Year
Bank balance	2000
over time (see	2001
Question 4)	2002
	2000

Year	Balance
2000	\$1,000
2001	\$1,100
2002	\$1,210
2003	\$1,331

To find the balance for 2001, we begin with the 2000 balance of \$1,000 and then add 10%:

Balance in 2001	=	balance in 2000 + 10% of balance in 2000
	=	\$1,000 + 10% of \$1,000
	=	\$1,000 + 0.10 × \$1,000 because 10% is 0.10
	=	1,000 + 100 = 1,100.

Similarly, to find the balance for 2002, we begin with the 2001 balance of \$1,100 and then add 10%. Notice that 10% of \$1,100 is not the same as 10% of \$1,000, and so this time the balance goes up by a different amount:

Balance in 2002	=	balance in 2001 + 10% of balance in 2001
	=	\$1,100 + 10% of \$1,100
	=	\$1,100 + 0.10 × \$1,100
	=	1,100 + 110 = 1,210.

Finally, to find the balance for 2003, we begin with the 2002 balance of \$1,210 and then add 10%:

Balance in 2003	=	balance in 2002 + 10% of balance in 2002
	=	\$1,210 + 10% of \$1,210
	=	\$1,210 + 0.10 × \$1,210
	=	\$1,210 + \$121 = \$1,331.

5. The formula for percent change – a very useful formula to know – is

Here, the level of methyl chloroform changes by 30 ppt (parts per trillion), dropping from its original level of 150 ppt to its current level of 120 ppt. Using our formula, we see that

Note that in general, to convert a decimal to a percentage, we shift the decimal point two places to the right. (Likewise, to convert a percentage to a decimal, we shift the decimal point two places to the left.) Here, we converted the fraction 30/150 to the decimal 0.20 by dividing:

second is given by

Ratio =

=

=

Recall that to divide by a fraction like 1/6, we must multiply by the fraction's reciprocal, which means that we flip the fraction "upside-down" and then multiply.

Percent change = <u>amount of change</u> original amount

Percent decrease = $\frac{\text{amount of decrease}}{\text{original amount}}$

 $=\frac{30 \text{ ppt}}{150 \text{ ppt}} = 0.20 = 20\%.$

 $\frac{30}{150} = \frac{1}{5} = 0.20 = 20\%.$

6. The fraction of never married Americans in 1990 is 1/4, and the fraction of never married Americans in 1970 is 1/6. The ratio of the first fraction to the

<u>1/4</u> 1/6	
$\frac{1}{4} \times \frac{6}{1}$	multiply by reciprocal
6/4	must be reduced

= 3/2 or 1.5.

7. Thinking in terms of proportionality, we see that the ratio of the stock's value today to its value one year ago (\$15,000) should equal the ratio of \$7,200 to \$6,000:

value today \$15,000	=	<u>\$7,200</u> \$6,000	
value today	=	\$7,200 × \$15,000	multiplying
	=	$\frac{6}{5}$ × \$15,000	reducing fraction
	=	6 × \$3,000	simplifying
	=	\$18,000.	

Thus, the investment would be worth \$18,000. Another way to solve this problem is to use our formula for percent change (see Question 5):

Percent change	=	amount of change original amount
	=	<u>\$7,200 - \$6,000</u> \$6,000
	=	$\frac{\$1,200}{\$6,000} = 0.20 = 20\%.$
Thus, if the person ha	ıd inv	vested \$15,000, it would grow by 20%:

\$15,000 + 20% × \$15,000 Value now = \$15,000 + 0.20 × \$15,000 because 20% = 0.20 = = \$15,000 + \$3,000 = \$18,000,

which is the same answer that we got before.

8. To evaluate these expressions, we find their values by "plugging in" the values of *v* and *w*.

(a) We have 3(v-2w) = 3((-2)-2(3))= 3(-2-6)= 3(-8) = -24.

(b) We have

Figure 1: For part (a) of

Question 9, we see

that the average

18-month-old girl

weighs 23 lbs and that the average

18-month-old boy

Question 9, we see

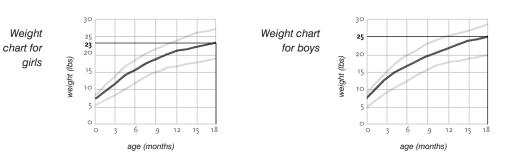
that a 5th percentile, 18- month-old boy

weighs the same as a 95th percentile,

6-month-old boy.

weighs 25 lbs.

Figure 2: For part (b) of



(b) The larger boy weighs 21 lbs at 6 months of age, and the small boy won't weigh this much until he is 18 months of age.

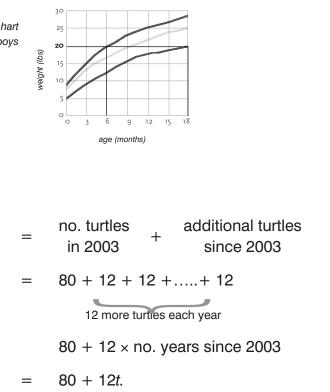
> Weight chart for boys

We have 10.

> no. turtles after t years

 $v^2 + w^2 = (-2)^2 + (3)^2$ = (-2)(-2) + (3)(3)= 4 + 9 = 13.

9. (a) The average 18-month-old girl weighs 23 pounds (lbs). The average 18-month-old boy weighs 25 lbs, or 2 lbs more than the girl.



Thus, a formula for P is P = 80 + 12t.

Another way to work this problem is to notice that the number of turtles is growing at a constant rate over time, which means that the equation for Pwill be linear, so that P = b + mt where b and m are constants. Here, b is the initial value, or 80, and *m* is the growth rate, or 12. This gives us $P = 80 + 10^{-10}$ 12t, the same answer that we got before.

11. (a) The perimeter of a shape is the distance around its border. The given shape has two unlabeled sides. From Figure 3, we see that these sides measure 2 and 4. Thus, adding up all the sides, we see that the perimeter of the shape is given by

Perimeter = 6 + 10 + 4 + 4 + 2 + 6 = 32 units.

(b) From Figure 4, we see that the shape can be broken into two different squares, one of side 6 and one of side 4. The area of a square is given by

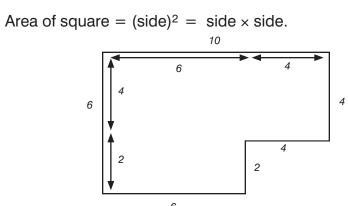


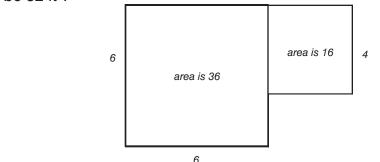
Figure 3: Finding the perimeter for part (a) of Question 11

> This means that the area of the square of side 6 is $6 \times 6 = 36$ squared units, and the area of the square of side 4 is $4 \times 4 = 16$ squared units, and so

> > area of shape = 36 + 16 = 52 squared units.

If the units were feet, the perimeter (a length) would be 32 ft. and the area would be 52 ft².

Figure 4: Finding the area for part (b) of Question 11



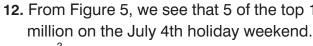
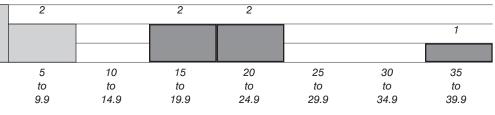


Figure 5: For Question 12, we see that 5 of the top 10 movies (darklv shaded) made more than \$10 million.



graph of cost versus distance will be a straight line.

0

to

4.9

would be 5, and we would have

Thus, the cost for driving 5 miles is \$31.10 at agency A but only \$14.00 at agency *B*. Agency *B* is cheaper for a short distance like 5 miles. On the other hand if we imagine driving 200 miles, the costs work out differently:

We see that to drive 200 miles would cost \$74 at agency A and \$92 at agency B. This means that to drive a long distance, agency A is cheaper.

costs equal to each other gives:

12. From Figure 5, we see that 5 of the top 10 movies made more than \$10

weekend receipts (\$ millions)

13. The formula for both agencies is *linear*. This means that for each agency a

(a) Suppose we imagine driving only 5 miles. In this case, the value of n

 $C_{\Delta} = 30 + 0.22 \times 5 = 31.10$

 $C_B = 12 + 0.40 \times 5 = 14.00.$

 $C_{A} = 30 + 0.22 \times 200 = 74$

 $C_B = 12 + 0.40 \times 100 = 92.$

(b) At what distance should we switch agencies? In other words, at what distance does agency A cost no more than agency B? We can answer this by solving the equation $C_A = C_B$. Setting the formulas for these two

> 12 + 0.40n = 30 + 0.22n0.40n - 0.22n = 30 - 120.18n = 18n = 18/0.18 = 100.

Thus, agency A will cost the same as agency B at n = 100 miles. This means that if we drive farther than 100 miles, we would save money by renting from agency A instead of agency B.

14. If a quantity increases by 100% it doubles in size. If it goes up by 200%, it triples in size; if it goes up by 300%, it quadruples in size; and so on. Since the number of injured skaters increased by 184%, this means that the number of injured skaters more than doubled - and almost tripled - in size.

15. We have

Total amount of money = amount for 1^{st} class + amount for coach.

Now, suppose (just for sake of argument) that 20 first-class tickets are sold for \$1,000 each. This would mean that

> Amount of money = \$1,000 per seat $\times 20$ seats = \$20,000. for 1st class

We see that to calculate the money brought in, we multiply the cost per seat by the number of seats. Since we aren't told the number of first-class seats or how much they cost, we must use the symbols N_f and P_f instead of 20 and \$1,000, but the reasoning is the same:

> Amount of money = P_f per seat × N_f seats = N_fP_f. for 1st class

Similarly, the amount of money for coach is given by

Amount of money = $\$P_c$ per seat $\times N_c$ seats = N_cP_c . for coach

Adding these two amounts together gives a formula for R, the total amount of money brought in by the airline for this flight:

R = amount for 1st class + amount for coach

$$= N_f P_f + N_c P_c \, .$$

and then multiply:

$$\frac{K_1}{K_2} = \frac{Z_1}{Z_2} \quad \text{original equation}$$
$$\frac{K_2}{K_1} = \frac{Z_2}{Z_1} \quad \text{flip both sides}$$
$$K \quad \frac{Z_2}{K_1} = \frac{Z_2}{Z_1} \quad \text{flip both sides}$$

$$K_2 = K_1 \frac{Z_2}{Z_1}$$
 or $= \frac{K_1 Z_2}{Z_1}$ multiply by K_1 .

Another approach is more direct. First, because we are solving for K_2 , we will multiply both sides by K_2 in order to clear it from the denominator:

$$\frac{Z_1}{Z_2} = \frac{K_1}{K_2}$$
 original equation

$$K_2 \quad \frac{Z_1}{Z_2} = K_1$$
 multiply by K₂.

$$\frac{Z_1}{Z_2} = \frac{K_1}{K_2} \qquad \text{original equation}$$
$$K_2 \quad \frac{Z_1}{Z_2} = K_1 \qquad \text{multiply by } K_2.$$

Next, divide both sides by the fraction Z_1/Z_2 in order to isolate K_2 :

To simplify this result, we recall that to divide by a fraction, we must multiply by its reciprocal. Since the reciprocal of Z_1/Z_2 is Z_2/Z_1 , we have:

17. Since one-fifth of the 0.6 grams (g) is given by

 $\frac{1}{5}$ ×

we see that 0.12 grams of powder spill from the dish. Thus, there are 0.60 - 0.12 = 0.48 grams left in the dish. Alternatively, since one-fifth of the 0.6 grams spills out, four-fifths of the 0.6 grams remain. This means that there are

16. One way to work this problem is to flip or invert both sides of the equation

$$K_2 = \frac{K_1}{Z_1/Z_2}$$
 .

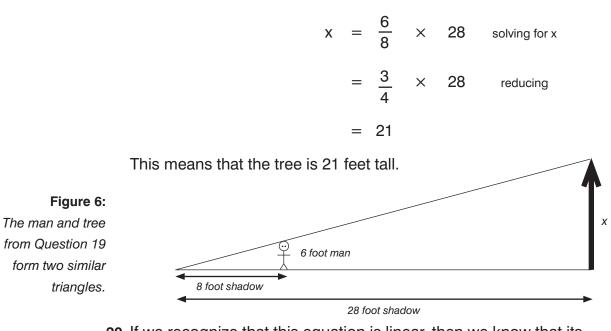
$$K_2 = K_1 \frac{Z_2}{Z_1} \, .$$

< 0.6 g =
$$\frac{0.6 \text{ g}}{5}$$
 = 0.12 g,

$$\frac{4}{5} \times 0.6 \text{ g} = \frac{2.5 \text{ g}}{5} = 0.48 \text{ grams left.}$$

by similar triangles

- 18. The scale is divided into 10 evenly spaced tick marks. This means that each small tick mark measures 0.20/10 = 0.02 units. Pointer A is at the third tick mark, which means it reads $3 \times 0.02 = 0.06$. Pointer B is at the seventh tick mark which means that it reads $7 \times 0.02 = 0.14$.
- **19.** From Figure 6, we see that the man, the tree, and the shadow form two similar triangles. In the figure, x stands for the height of the tree, which is what we would like to determine. Since the triangles are similar, the ratios of corresponding sides are equal. In other words, the ratio of the tree's height to its shadow's length, x/28, equals the ratio of the man's height to his shadow's length, 6/8. This gives us the equation x/28 = 6/8, which we can solve for x:



 $\frac{x}{28} = \frac{6}{8}$

20. If we recognize that this equation is linear, then we know that its graph will be a straight line. We can find the s-intercept by setting t = 0, which gives:

$$5s + 15 - 0 = 0$$
 setting $t = 0$
 $5s = -15$

Similarly, we can find the t-intercept by setting s = 0, which gives:

Thus, the s-intercept is the point (t, s) = (0, -3) and the t-intercept is the point (t, s) = (15, 0). Plotting these two points, we can draw a line passing through them to find the graph of the equation. (See Figure 7.)

Figure 7: The graph of the equation from Question 20 is a straight line.

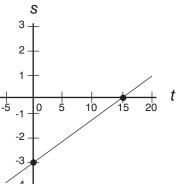


Another approach is to first place this equation into slope-intercept form in other words, to write it as s = mt + b where m is the slope and b is the *s*-intercept (the vertical intercept). Solving for *s* gives

5s

To find the *t*-intercept, we set s = 0 and solve. We obtain t = 15, as before.

- s = -3.
- 0 - t = setting s = 0
- 15 t = 0
 - t = 15.



$$+ 15 - t = 0
5s + 15 = t
5s = t - 15
s = \frac{1}{5}(t - 15)
= \frac{1}{5}t - 3.$$

Thus (as we have already seen) the *s*-intercept is b = -3. The slope is $m = \frac{1}{2}$ 5

21. (a) We have
$$(2 \times 10^{-4}) (3 \times 10^{5}) = 2 \times 3 \times 10^{-4} \times 10^{5}$$

= $6 \times 10^{-4+5}$
= 6×10^{1} ,

which is the same as 60.

(b) We have
$$(3 \times 10^{-4})(8 \times 10^{5})$$

 4×10^{-7} = $3 \times 8 \times 10^{-4} \times 10^{5}$
 4×10^{-7}
= $24 \times 10^{-4+5}$
 4×10^{-7}
= $8 \times \frac{10^{-1}}{10^{-7}}$
= $6 \times 10^{1-(-7)}$
= 6×10^{8} ,

which equals 600,000,000 or 600 million.

22. Equations (i) and (iii) are both linear. The slope of (i) is -50, so this population decreases (goes down) by 50 animals per year. The slope of (iii) is +70, so this population increases by 70 animals per year. Both populations start out at the same level: 1,000. We can see this by setting *t* equal to 0 in these equations.

On the other hand, equations (ii) and (iv) are not linear. Equation (ii) is exponential, and equation (iv) is sinusoidal. If we try several different values for t in equation (ii), such as t = 0, 1, 2, and 3, we see that this population goes down by 5% each year:

8000(0.95) ⁰	=	8000	
8000(0.95) ¹	=	7600	a 5% decrease
8000(0.95) ²	=	7220	a 5% decrease
8000(0.95) ³	=	6859	a 5% decrease.

By process of elimination, we see that (d) must go with equation (iv). This makes sense, because sinusoidal quantities rise and fall over time.

Putting all of this together, we see that statement (a) goes with equation (ii), statement (b) goes with equation (iii), statement (c) goes with equation (iv), and statement (d) goes with equations (i) and (iii).

b, determines where the line crosses the y-axis.

Since equations (a) and (b) have the same y-intercept (b = 6), they must cross the y-axis at the same point. Moreover, equation (a) has a negative slope (-0.8) while equation (b) has a positive slope (+0.6). Notice that lines A and *D* have the same *y*-intercept and that line *A* climbs while line *D* falls. Thus line A corresponds to equation (b) while line D corresponds to equation (a).

On the other hand, equations (c) and (d) have the same slope (+0.5), and thus they describe lines of the same steepness, which is to say they describe parallel lines. Moreover, the y-intercept of equation (c) is less than 6, while the y-intercept of equation (d) is more than 6. This is significant because lines A and D cross the y-axis at 6. Thus, we know that equation (c) describes a line that crosses the y-axis below lines A and D. Similarly, equation (d) describes a parallel line that crosses the y-axis above lines A and D. Thus, line C corresponds to equation (c) while line B corresponds to equation (d).

times) the rate is positive.

23. The key to this problem is to understand what the values of *m* and *b* tell us about the graph of the linear equation y = mx + b. A positive value of the slope *m* corresponds to a line that rises when read from left to right, while a negative value of *m* corresponds to a line that falls. The larger the value of *m*, either positive or negative, the steeper the line. The value of the y-intercept,

24. It is important to realize that the graph does not show the amount of water in the reservoir; rather, it shows the *rate* that water is entering or leaving the reservoir. We are told that water is entering the reservoir when the rate is positive, which means that the volume is increasing when the rate is positive. Thus, the answer is time 0 to time C, because on this interval (and at no other

26 • Quantitative Reasoning Assessment

STOP!!!

- Don't start either assessment until you are ready.

- the actual assessment exam.)
- on page 43. Check these first.
- starting on page 47.

These are actual QR Assessments, given to students in 1995 and 1999. They are similar to the one you will take upon arrival at the College.

• First, solve the study problems (starting on page 3).

• Allow 90 minutes to finish each practice assessment.

• Do not use notes or a calculator. (Calculators may not be used during

Short answers for both practice assessments have been provided starting

• Worked-out solutions for the 1995 assessment have also been provided,

• For each of these assessments, a student who answered more than 9 out of 18 questions correctly was considered to have passed.

• *Tip:* Work each practice assessment and then check the short answers (starting on page 43). If you miss a question, try to figure out how to work it on your own before you read the solution. You may feel the urge to read the solutions right away, but discovering the answer to a problem on your own is usually a much more valuable learning experience.

Practice Exam: QR Assessment for 1995

- drop during this time period?

- and c- intercepts.

Figure 1: Use these axes to answer Question 4.

is the object next to the ruler?

Figure 2:

For Question 5, object.

0 in

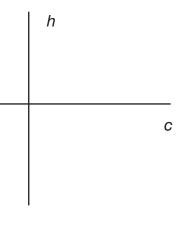
measure the length (in inches) of the

1. The number of tuberculosis cases in New York state dropped from 4,000 in 1993 to 3,600 in 1994. By what percent did the number of tuberculosis cases

2. A rectangular swimming pool is 30 meters long, 10 meters wide, and 3 meters deep. Assuming that 1,000 cubic centimeters of water weighs two pounds, how much does the water in a full pool weigh?

```
3. Evaluate the expression 5(3q^2 - 4p) assuming that p = -3 and q = -5.
```

```
4. Graph the equation 3h + 5c = 30 on the set of axes provided. Label the h-
```

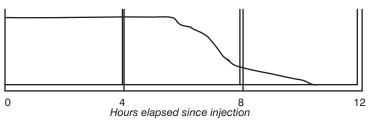


5. The figure shows a 1/4-inch long segment of a ruler. How many inches long

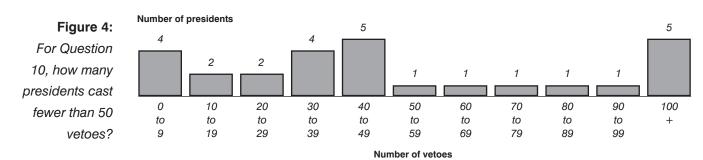


6. A patient is given an injection of a therapeutic drug. Over time, the level of the drug in the body falls as the drug is metabolized. Figure 3 shows the level of the drug in the body over a 12-hour period. During which of the three 4-hour periods shown:

Figure 3: The drug level over a 12-hour period (see Question 6)

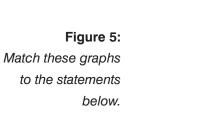


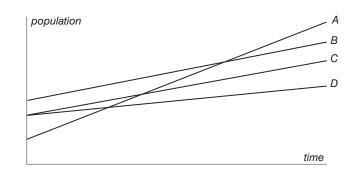
- does the drug level drop by the greatest amount? (a)
- (b) is the drug level most nearly constant?
- 7. The total cost of planting a crop depends on two things: the cost of the equipment, which is fixed, and the cost of seed, fertilizer, irrigation, and labor, which depends on the number of acres sown. Suppose a farmer must spend \$120,000 on equipment plus \$3,500 for every acre sown. Find a formula for *C*, the farmer's total cost, in terms of *n*, the number of acres sown.
- 8. A beaker contains 83.2 milliliters (ml) of liquid. The liquid is heated until one-fourth of it has evaporated. How much liquid (in ml) remains in the beaker?
- 9. Let $U_A = \frac{r}{1 U_B}$. Solve this equation for U_B in terms of U_A and r.
- 10. Figure 4 gives the number of U.S. presidents who cast a given number of vetoes during their terms in office. For example, we see from the figure that only one president cast between 60 and 69 vetoes during his term in office. (It happened to be Gerald Ford with 66 vetoes.) Based on the chart, how many presidents cast fewer than 50 vetoes during their terms in office?



- committed there?

- correspond to no matching verbal statement.





- (a)
- (b)

11. The *hate-crime rate* of a state is the ratio of hate or bias-related crimes per year to the number of people living in the state. According to The New York Times, in 1994, New Jersey's hate-crime rate was 13 hate crimes per 100,000 residents. Assuming that in 1994 the population of New Jersey was eight million people, how many hate-related crimes were

12. A parcel of land measures 2 3/4 acres. A developer wishes to divide the land into lots for houses for each lot measuring 1/7 of an acre. Into how many complete lots can the acres of the parcel of land be divided?

13. Let W represent the number of female employees and M the number of male employees at a certain company. Write an expression in terms of W and M that represents the fraction of employees that are women.

14. The graphs in Figure 5 describe the growth of four different populations (labeled A, B, C, and D) over an extended period of time. For each of the verbal statements following the figure, indicate the two populations that it best describes. Note that some of the populations shown in the figure may

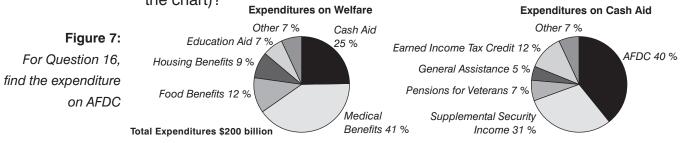
These two populations begin with the same number of members.

These two populations grow at the same rate.

- 15. As shown in Figure 6, a man whose eyes are six feet above the ground stands next to a round pit dug into the ground. The pit measures five feet cross. How deep is the pit (in feet)?
- Figure 6: For Question 15. find the depth of the pit

Note: figure NOT drawn to scale nit 2 ft 5 ft

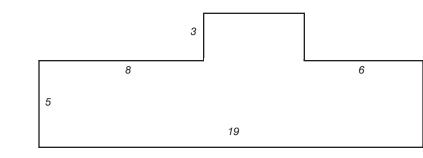
16. Based on the charts shown in Figure 7, how much money (in dollars) was spent on Aid to Families with Dependent Children (labeled AFDC on the chart)?



- 17. A commercial artist needs to rent some high quality photographic equipment to reproduce her artwork. She considers two different types of equipment. The cost of renting and using the first type is \$400 plus \$2.50 per copy. The cost of renting and using the second type is \$150 plus \$5.00 per copy.
 - (a) Which type of equipment is less expensive if the artist only needs to make a small number of copies, the first or the second?
 - (b) How many copies would the artist need to make before the other type of equipment becomes less expensive?

18. Find the perimeter and area of the shape in Figure 8.

Figure 8: For Question 18. find the perimeter and area of this shape.



Practice Exam: QR Assessment for 1999

Figure 1:

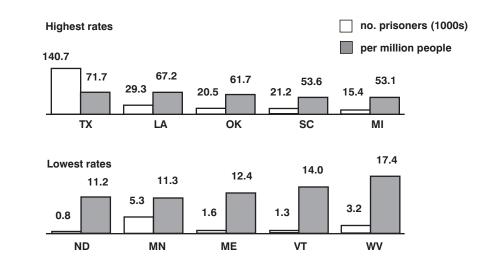
States with the

highest rates (top

chart) and lowest

rates (bottom chart)

of prisoners in 1997



is it?

- other nine states shown by the figure.
- (ND) that is in prison.
- twice the rate (per million) in Maine.
- (VT).

1. Figure 1 shows the states with the highest and lowest rates of prisoners.¹ The figure shows both the total number of prisoners (in 1,000s) in each state and the prison rate per million people in the population. For instance, we see from the figure that in Vermont (VT), there is a total of 1,300 prisoners, which works out to 14 prisoners for every million people in the population.

One of the statements (a)-(d) is not supported by Figure 1. Which statement

(a) There are more prisoners in Texas (TX) than the combined total of the

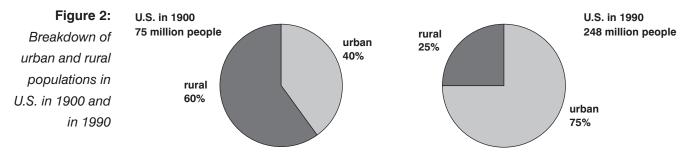
The percentage of the population that is in prison in Minnesota (MN) is more than five times the percentage of the population of North Dakota

(c) Although there are twice as many prisoners in West Virginia (WV) as there are in Maine (ME), the rate (per million) in West Virginia is less than

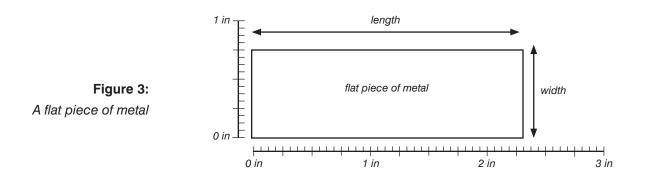
(d) The state with the smallest number of prisoners is North Dakota (ND), and the state with the next smallest number of prisoners is Vermont

¹Data are from The New York Times, August 9, 1998. Data are for 1997.

2. Figure 2 shows the breakdown of urban and rural populations in the U.S. in 1900 and in 1990.



- (a) How large was the rural population of the U.S. in 1900?
- (b) The rural population of the U.S. actually grew in size between 1900 and 1990, even though it went down in proportion to the overall population. By how many people did the rural population grow?
- 3. Figure 3 gives an enlarged view of a flat piece of metal.



- (a) What is the length of the piece of metal in inches (in)?
- (b) What is the area of the piece of metal in square inches (in²)?
- 4. A hectare is a metric unit of area and an acre is a U.S. unit of area. Both are often used to measure the sizes of large plots of land.
 - (a) A hectare is equivalent in area to a square plot of land measuring 100 meters by 100 meters. Suppose a certain piece of land measures 4 km by 8 km. What is its area in hectares? Note: Recall that 1 kilometer (km) equals 1,000 meters.

- - 1 billion equals 1,000 million.
- her enough time for 18 client meetings.

week week 1 s, no. staff 6 meetings c, no. client 18 meetings

Table 1:

Weekly meeting

corporate executive

schedule of a

Let c stand for the number of client meetings and s stand for the number of staff meetings. Find a *simplified* formula for *c* in terms of *s*.

(b) An acre is approximately equivalent to 1/640 of a square mile. One hectare equals two and a half acres. A certain piece of land measures 5 miles by 5 miles. What is its area in hectares?

5. The national debt is the amount of money owed by the U.S. government. As of July 1998, the U.S. national debt amounted to \$5.54 trillion.

(a) Suppose the debt were reduced by \$120 billion. What would the new debt be? Note: In the U.S., 1 trillion equals 1,000 billion.

(b) As of July 1998, the size of the U.S. population was 270 million. Suppose every person in America contributed \$5,000 towards paying off the national debt. What would the remaining balance be? Note: In the U.S.,

6. A busy corporate executive divides her time at the office each week between meetings with staff and meetings with clients. Each staff meeting takes 4 hours, and each client meeting takes two hours. The number of staff meetings changes from week to week, and she must attend all of them. She works a full 60 hours every week, and every hour not spent at a staff meeting is spent meeting with clients. Table 1 shows the number of staff and client meetings each week over a six-week period. For example, we see that during week 1 the executive has six staff meetings. In a 60-hour week, this leaves

week 2	week 3	week 4	week 5	week 6
5	9	4	7	15
20	12	22	16	0

7. According to the *logistic model* of population growth, an initially small population will grow rapidly at first but then will level off at a maximum value (the so-called *carrying capacity*). The size of the population in year t is given by the formula

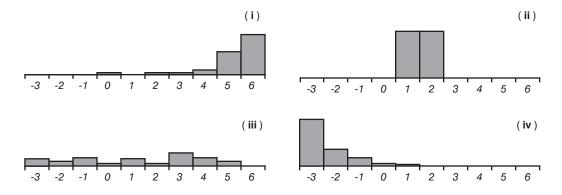
Population
in year t =
$$\frac{L}{1 + k (1/2)^{t/\tau}}$$

where L is the carrying capacity and where k and τ are constants. Suppose that for a certain population of insects, L = 4500, k = 8, and $\tau = 3$.

- (b) How large is this population initially (in year t = 0)?
- (c) How large is this population 9 years later (in year t = 9)?
- 8. When studying sedimentary deposits, geologists are often interested in the distribution of grain size – that is, the proportions of different sizes of grains there are in a given deposit. To measure grain size, geologists use what is known as the ϕ grade scale, where ϕ is a Greek letter pronounced either fee or fie. Numbers on this scale range from -12 for very coarse material (like boulders) to +14 for very fine material (like clay). A zero (0) on the scale represents coarse sand. Figure 4 shows grain-size distrib- utions for four different samples of sediment, numbered (i) through (iv).



Grain-size distributions for four different samples. The horizontal axis shows grain size using the ♦ grade scale. The columns represent the proportion of each sample consisting of a given grain size.



Match each of the following statements to the grain-size distribution from Figure 4 it best describes.

- (a) This sample is taken from a sandy beach with no rocks. The sand grains range in size from coarse to fine.
- (b) This sample is taken from a laboratory sieve. All of the coarsest and finest grains have been filtered out, leaving a very uniform mix.

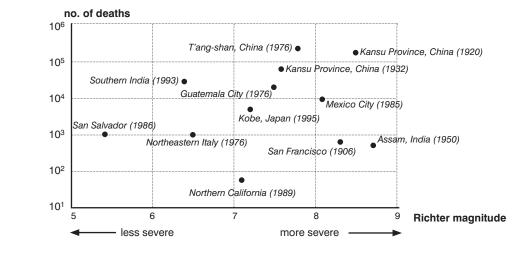
the quake.

Figure 5:

the world

Earthquakes this

century from around



- as the San Salvador earthquake in 1986?
- **Note:** Your answer will be an algebraic expression.

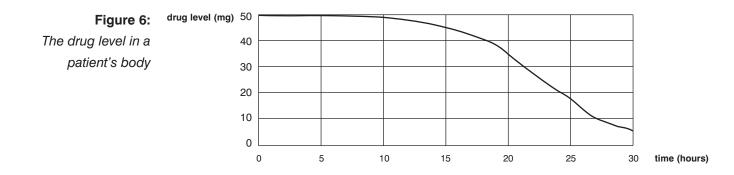
9. Figure 5 describes twelve earthquakes that occurred in different parts of the world. The horizontal axis gives the magnitude of the earthquake using the Richter scale. The vertical axis gives the number of people killed by

(a) How many earthquakes were less severe than the Mexico City earthquake in 1985, and yet killed more people?

(b) How many of the earthquakes killed at least 100 times as many people

10. A general contractor is hired to complete an addition to a single-family house. The contractor does much of the work herself, but hires a plumber and electrician for certain parts of the job. The contractor charges p_1 per hour and bills a total of n_1 hours. She pays the plumber p_2 for n_2 hours of work and the electrician p_3 per hour for n_3 hours of work. How much money does the contractor keep after paying the plumber and the electrician?

11. A patient is given an injection of a therapeutic drug. Over time, the level of the drug in the body falls as the drug is metabolized. Figure 6 shows the level of the drug in the body (in milligrams, or mg) over a 30-hour time period.



(a) Which of the following drops in drug level takes the most time?

From 50 mg	From 45 mg	From 40 mg	From 30 mg	From 20 mg
to 45 mg	to 40 mg	to 30 mg	to 20 mg	to 10 mg

(b) On average, about how fast (in mg/hr) does the drug level drop between hour 20 and hour 30?

2 3 20 30 40 50 1 5 10 4

- 12. Nantucket, a small island off the coast of Massachusetts, is the smallest county in the state. Wellesley is in Norfolk County, the fifth largest county in the state.
 - (a) Over the past few years the population of Nantucket has grown rapidly. In 1990, its population was 6,000, but by 1996 its population had grown to 7,200. Suppose the population of Nantucket goes on to increase by the same proportion between 1996 and 2002 as it did between 1990 and 1996. How large will it be in 2002? Note: The population is assumed to increase by the same *proportion*—not by the same number of persons.

13. Let *E* stand for the fuel efficiency of a car, in miles per gallon (mpg).

- fuel efficiency.
- *E* to be 42.5?
- heat energy have been added.
- four populations are

 P_2

 P_2

where *L* and *r* are positive constants. For each of the following verbal descriptions, determine which of the populations of the statement could describe. Note: Some statements may describe none of the populations, and some may describe more than one.

(a) This population begins (in year t = 0) at the same level as P_{1} .

 P_3

(b) This population grows at the same annual rate as P_1 .

 P_3

(b) In 1996, the population of Norfolk County was 639,000. Suppose the population of Norfolk County had increased by the same proportion between 1990 and 1996 as Nantucket's population did. How large would Norfolk County's population have been in 1990?

(a) Suppose the car travels for 4 hours at a rate of 60 mph, and that it burns 8 gallons of gas during this time. Find the value of *E*, the car's

(b) How far must the car travel on 8 gallons of gas in order for the value of

14. If 1 calorie of heat energy is added to a 1 gram (g) piece of copper, the temperature of the copper will go up by 11°C. The temperature of a 1 g piece of copper is 25°C. Find a formula for its temperature, T, after Q calories of

15. Four different animal populations are increasing in size as time goes by. Each population grows at a constant annual rate. Let P_1 , P_2 , P_3 , and P_4 stand for the sizes of the four populations in year t. Formulas for these

> ₁	=	L + rt
2 2	=	3L + 2rt
> 3	=	L/2 + (r + 250) t
5 4	=	L + 250 + <i>rt</i>

 P_4

No match

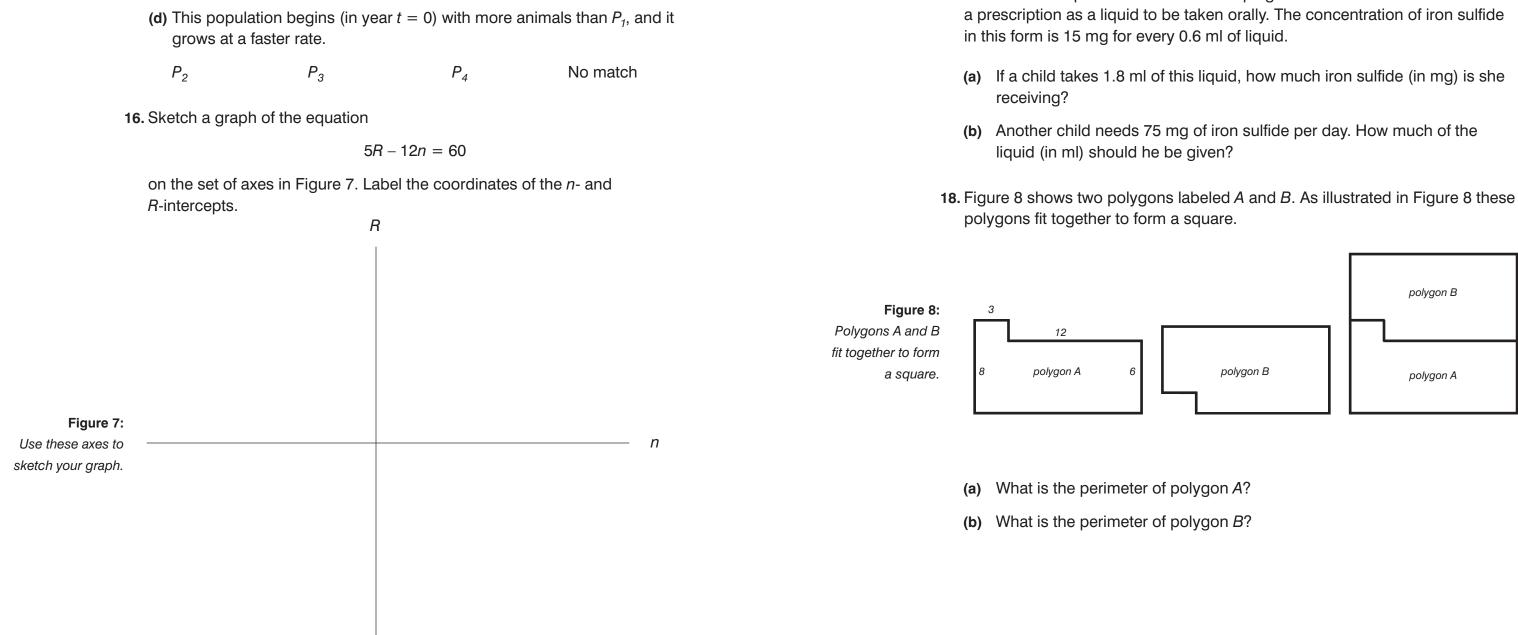
 P_2

(c) This population grows by 250 more animals per year than P_1 .

 P_4

No match

 P_3



17. Children with elevated lead levels in their blood are typically given high doses of iron (in the form of iron sulfide), because iron displaces lead from vulnerable receptors in a child's developing brain. Iron sulfide is sold without

42 • Quantitative Reasoning Assessment

STOP!!!

- next page.
- starting on page 47.
- valuable learning experience.

• Don't read these solutions until you have tried the problems on your own!

• Short answers for both practice assessments are provided beginning on the

• Worked-out solutions have also been provided for the 1995 assessment,

• *Tip:* Solve the study problems and then check the short answers. If you miss a question, try to figure out how to work it on your own before you read the answer. You may feel the urge to read the answers right away, but discovering how to solve a problem on your own is usually a much more

Short Answers to QR Assessment for 1995

Short Answers to QR Assessment for 1999

1. 10%		1.	statement (b)
2. 1,800,000 lbs		2.	(a) 45 million
3. 435		3.	(a) 2 5/16 inches, or 37/16 inc
4. The <i>c</i> -intercept is 6 and the h -	-intercept is 10.	4.	(a) 3,200
5. 9/64 inches		5.	(a) \$5,420 billion or \$5.42 trill
6. (a) 2 nd 4-hour period	(b) 1 st 4-hour period	6.	c = 30 - 2s
7. $C = 120,000 + 3,500n$		7.	(a) 500
8. 62.4 ml		8.	(a) histogram (i)
9. $U_B = 1 - r/U_A$		9.	(a) 4
10.17 presidents		10	$p_1n_1 - p_2n_2 - p_3n_3$
11. 1,040 crimes		11	. (a) from 50 mg to 45 mg
12.19 complete lots		12	. (a) 8,640
13. <i>W</i> /(<i>W</i> + <i>M</i>)		13	. (a) 30
14. (a) C and D	(b) <i>B</i> and <i>C</i>	14	. <i>T</i> = 25 + 11 <i>Q</i>
15. 15 feet		15	. (a) no match
16. \$20 billion (\$20,000,000,000)			(c) P ₃
17. (a) 2 nd type	(b) more than 100 copies		

18. (a) Perimeter is 54 units. (b) Area is 110 squared units.

(h)	17	mil	lion
(u)	17	11111	IIOII

inches (b) 1 47/64 in², or 111/64 in²

(b) 6,400

rillion (b) \$4,190 billion or \$4.19 trillion

(b) 2,250

(b) histogram (ii)

(b) 2

(b) 3

(b) 532,500

(b) 340

(b) P₄ (d) P₂ **16.** See Figure 1: solution to Question 16.

Worked Solutions to QR Assessments for 1995

Here, the number of tuberculosis cases dropped by 400, from 4,000 to 3,600. This means that the number of tuberculosis cases dropped by

2. The volume of the pool is given by

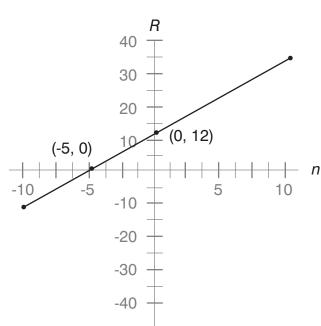
We aren't told how much a cubic meter (m³) of water weighs. However, we know that 1000 cubic centimeters (cm³) weigh about 2 pounds (lbs). This means that we should convert our volume from m³ to cm³:

m³ to cm³.

This gives (counting zeros)

Since 1,000 cm³ weighs about 2 lbs, we have

3. We have



17. (a) 45	(b) 3
18. (a) 46 units	(b) 48 units

1. The formula for percent change – a good formula to memorize – is

Percent change = $\frac{\text{amount of change}}{\text{original amount}}$.

Percent decrease = $\frac{\text{amount of decrease}}{\text{original amount}} = \frac{400}{4000} = 10\%$.

Volume = length \times width \times depth

= 30 m × 10 m × 3 m = 900 m³.

Volume (in cm³) = 900 m³ × $\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^{3}$.

Notice that we must multiply by three factors of 100 to convert from

Volume = $900 \times 100 \times 100 \times 100 = 900,000,000 \text{ cm}^3$.

Weight = 900,000,000 cm³ × $\frac{2 \text{ lbs}}{1000 \text{ cm}^3}$ = 1,800,000 lbs.

 $5(3q^2 - 4p) = 5(3(-5)^2 - 4(-3))$

= 5(3(25) + 12)

= 5(87) = 435.

4. This equation is linear, so its graph will be a line. One way to work this problem is to find the axis intercepts and use them to draw the line. To find the *h*-intercept, we set c = 0 and solve:

> 3h + 5(0) = 303h = 30h = 10.

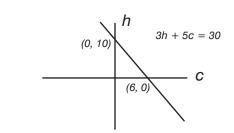
Thus, the point (c,h) = (0, 10) is on the line. We now find the *c*-intercept:

$$3(0) + 5c = 30$$

 $5c = 30$
 $c = 6.$
8. One-fourth of the liquid, or
 $(1/4) \times 82.2 = 20.8$ ml of liquid

Thus, the point (c, h) = (6, 0) is on the line. (See Figure 1.)

Figure 1: The graph of the equation in Question 4 is a straight line.

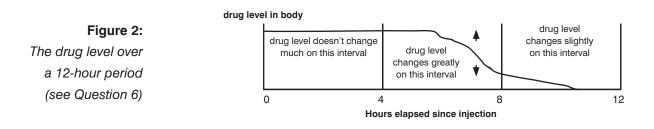


5. The ruler shows a 1/4-inch length divided into 16 smaller units. The length of each of these units is

Length =
$$1/4$$
 inch $\div 16 = \frac{1/4}{16} = 1/64$ in.

The length of the mark is 9 of these units. Thus, the length of the mark is $9 \times$ 1/64 = 9/64 inches.

6. See Figure 2.



(a) The drug level drops by the greatest amount during the second 4-hour period, so it is falling most rapidly on this time interval.

(b) The drug level hardly drops at all during the first 4-hour period, so it is almost constant on this time interval.

7. We have

Total cost = cost for equipment + cost per acre \times no. acres

which aives

amount remaining is 83.2 - 20.8 = 62.4 ml.

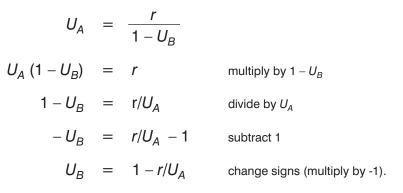
9. We have

Number of presidents

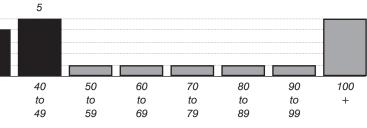
5 2 2 0 0 10 20 30 to to to to 9 19 29 39

Figure 3: For Question 10, the darker columns show those U.S. presidents casting fewer than 50 vetoes 0 + 3,500n.

one-fourth of 83.2 ml, evaporates. This amounts to $(1/4) \times 83.2 = 20.8$ ml of liquid. Since this is the amount that evaporates, the

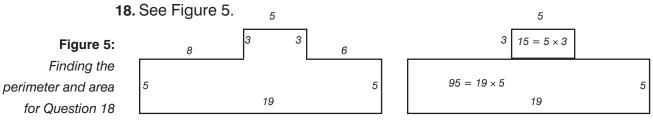


10. The darker columns in Figure 3 indicate those presidents who cast fewer than 50 vetoes. From Figure 3, we see that in total there are 4 + 2 + 2 + 4 + 5 = 17 presidents in this category.

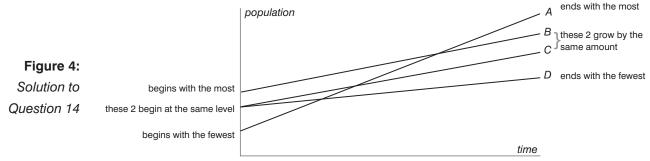


Number of vetoes

(b) We also see that populations B and C grow at the same rate, because 11. We have no. hate crimes in NJ 13 crimes no. people in NJ 100,000 persons lines *B* and *C* are parallel. Since there are 8 million people in New Jersey, we have **15.** Letting x stand for the depth of the pit, we have (by similar triangles) no. hate crimes in NJ 13 crimes <u>x</u> 5 6 8,000,000 persons 100,000 persons 8,000,000 persons × 13 crimes Х = 15. no. hate crimes in NJ = 100,000 persons 16. From the chart, we see that \$200 billion was spent on welfare. Of this \$200 = $13 \text{ crimes} \times 80 = 1040 \text{ crimes}.$ billion, 25%, or \$50 billion, was spent on cash aid. And of this \$50 billion, 40% was spent on Aid to Families with Dependent Children (AFDC). Since 12. There are 2 3/4 acres, which can be written as 11/4 acres. To find out how 40% of \$50 billion is \$20 billion, this means that \$20 billion was spent on many 1/7-acre lots will fit on this land, we must divide 11/4 by 1/7: AFDC. $\frac{11/4}{1/7} = \frac{11}{4} \times \frac{7}{1} = \frac{77}{4} = 19.25.$ **17. (a)** To make 1 copy using the first type of equipment costs 400 + 2.50= \$402.50. To make 1 copy using the second type of equipment costs This means that 19 complete lots can be built, leaving 0.25 (1/4 acre) 150 + 5 = 155. Clearly, the second type of equipment is cheaper to undeveloped. use than the first type, provided that only a few copies are needed. 13. The total number of women is W and the total number of employees is (b) We can answer this question by finding the value of *n* where the cost for W + M. We have the first method equals the cost for the second method. This gives 400 + 2.50n = 150 + 5.00n= <u>no. of women</u> = <u>W</u> Fraction of employees that are women W + M250 = 2.50n14. See Figure 4. n = 100.ends with the most population This means that both methods cost the same amount for n = 100. We know these 2 grow by the from part (a) that the second method is cheaper for just a few copies. Here same amount we see that if n > 100, then the first method becomes less expensive.



(a) The perimeter is 54 units.



(a) From the figure, we see that populations C and D begin with the same number of members, because lines C and D start at the same point on the vertical axis.

(b) The area is 110 squared units.