Solutions - Practice QR Assessment 2

1) **Answers:** b, d, and e

**Solution:** This is a problem about working with scientific notation.

Option a is false. \(10^6\) is one million, so \(6.9 \times 10^6\) is 6.9 million, not 69 million

Option b is true. \(5.8 \times 10^8 - 3.3 \times 10^8 = (5.8-3.3) \times 10^8 = 2.5 \times 10^8\), and \(10^8\) is 100 million, so the population of North America minus the population of the United States is \(2.5 \times 100\) million = 250 millions

Option c is false. (You cannot subtract numbers in scientific notation, like \(7.8 \times 10^9 - 2.9 \times 10^4\) by subtracting the exponents.)

\[
7.8 \times 10^9 - 2.9 \times 10^4 = 780,000 \times 10^4 - 2.9 \times 10^4 \\
= 779,997.1 \times 10^4 \\
= 7.779971 \times 10^9 \\
\text{This rounds to } 7.8 \times 10^9 .
\]

Option d is true. To calculate this, you need to divide the population of the world by the population of Wellesley:

\[
\frac{7.8 \times 10^9}{2.9 \times 10^4} = \frac{7.8}{2.9} \times 10^{9-4} = 2.7 \times 10^5
\]

There are about 270,000 times as many people on earth as there are living in Wellesley.

Option e is true. \(10^9 = 1\) billion. The population of the world is 7.8 billion, or just under 8 billion
2) **Answer:** a

**Solution:** This is a problem about histograms and medians. First, you need to know how many presidents are represented on this table, by looking at the heights of the columns. There are 16 who had 0-9 vetoes, 7 who had 10-19, and so on. Add up the heights of all of the columns to see that there are 44 presidents. The median value is the average of the number of vetoes from the presidents who issued the 22nd and 23rd fewest number of vetoes. There were 16 who issued 0-9, and another 7 -- presidents numbers 17 through 23 -- who issued 10-19. As the 22nd and 23rd fewest number of vetoes both landed in the 10-19 column, the median is a: 10-19.
3) **Answer:** 1,040 hate crimes

**Solution:** The hate crime rate of 13 per 100,000 means:

\[
\frac{\text{number of hate crimes in New Jersey}}{\text{population of New Jersey}} = \frac{13 \text{ crimes}}{100,000 \text{ people}}
\]

As there are 8 million people in New Jersey, we know:

\[
\frac{\text{number of hate crimes in New Jersey}}{8,000,000} = \frac{13}{100,000}
\]

Solve for the number of hate crimes in New Jersey to get the answer:

\[
\text{number of hate crimes} = \frac{13}{100,000} \times 8,000,000 = 1,040
\]

There were 1,040 hate crimes in New Jersey in 1994.
4) **Answer:** 19 minutes

**Solution:** When the water in the bathtub is 18 inches deep, the total volume of water will be $60 \times 20 \times 18 \text{ in}^3$, or 21,600 in$^3$. This needs to be converted into gallons, with the following calculation:

$$21,600 \text{ in}^3 \times \frac{1 \text{ ft}^3}{12^3 \text{ in}^3} \times \frac{7.5 \text{ gallons}}{1 \text{ ft}^3} = \frac{21,600 \times 7.5}{1,728} \text{ gallons}$$

The bathtub holds 93.75 gallons of water when full. As 5 gallons flow in per minute, it will take \( \frac{93.75 \text{ gallons}}{5 \text{ gallons per minute}} \) or 18.75 minutes to fill the bathtub. Rounding to the nearest whole number gives the answer, 19 minutes.
5) **Answer:** a

**Solution:**

a) There are 1,981,503 in Colorado with a Bachelor's and 368,746 in West Virginia. Colorado has \( \frac{1,981,503}{368,746} \), or over 5, times as many people with a Bachelor's as West Virginia does: (a) is false.

b) There are 1,372,453 in Connecticut and 632,034 in Mississippi. \( \frac{1,372,453}{632,034} = 2.17 \), which is around 2: (b) is true.

c) This is a statement about rates, not numbers. For every 1,000 people, 421 in Massachusetts have Bachelor's and 234 in Louisiana do. Divide those numbers: \( \frac{421}{234} = 1.80 \). Subtract 1 to find how much more likely people in Massachusetts are to have a Bachelor's: \( 1.80 - 1 = 0.80 \), or 80%. Yes, people in Massachusetts are 80% more likely to have Bachelor's than people in Louisiana: (c) is true.

d) There are 3,349,712 in New Jersey with a Bachelor's, and 641,502 + 1,006,733 + 1,060,809 = 2,709,044 in Arkansas, Kentucky, and Louisiana combined: (d) is true.

e) New Jersey has 3,349,712 people with Bachelors', and Maryland has 2,251,685, so the first statement is true. The second statement is comparing the rates in the two states. In New Jersey, 381 people out of every 1,000 have Bachelor's, whereas in Maryland 390 people out of every 1000 do -- a higher rate. People in Maryland are more likely to have Bachelor's: (e) is true.
6) **Answer:** 14 (million people)

**Solution:**
In 1900, the rural population was 60.4% of 75 million, or \(0.604 \times 75 \text{ million} = 45.3 \text{ million}\).

In 2000, the rural population was 21.0% of 282 million, or \(0.21 \times 282 \text{ million} = 59.2 \text{ million}\).

The rural population grew by \((59.2-45.3) \text{ million}, or 13.9 \text{ million people. Round to the nearest million to get the answer, 14 million people.}\)
7) **Answer:** 6,400 hectares

**Solution:** The area of the land is $5 \times 5 = 25 \text{ miles}^2$. Convert this to hectares as follows:

\[
25 \text{ miles}^2 \times \frac{640 \text{ acres}}{1 \text{ miles}^2} \times \frac{1 \text{ hectare}}{2.5 \text{ acres}} = 6,400 \text{ hectares}
\]
8) **Answer:** 15

**Solution:** From the description of the problem, you can write the equation $45 = 5s + 3m$. Now simplify this to get an equation for $s$:

\[
45 = 5s + 3m \\
45 - 3m = 5s \\
\frac{45 - 3m}{5} = s \\
15 - \frac{3m}{5} = s \\
15 - 0.6m = s
\]
9) **Answer:** 2

**Solution:** The San Salvador earthquake killed approximately $10^3$, or 1,000, people. An earthquake that killed at least 100 times as many people would need to kill at least $10^5$ people. There are two earthquakes on this chart above the line that represents $10^5$ deaths: the T’ang-shan, China quake and the Kansu Province, China quake.
10) **Answer:** 16 ml

**Solution:** How many 5 ml doses does the child need? To get 500 mg of analgesic, the child needs $\frac{500}{160} = 3.125$ doses of the medicine, as each dose has 160 mg in it. The amount of liquid in 3.125 doses is $3.125 \times 5$, as there are 5 ml per dose. The child needs 15.625 ml of liquid, which rounds to 16 ml.
11) **Answer:** 34 degrees

**Solution:** The temperature of coffee is decreasing exponentially by 8%. Its temperature will be $180(0.92)^t$ after $t$ minutes. After 20 minutes, this equation gives the answer, $180(0.92)^{20} = 33.96$ degrees, or, to the nearest degree, 34 degrees.
12) **Answer:** c

**Solution:**

Call the original price $P$. Then

\[
P - 0.15P = 70 \\
0.85P = 70
\]

Divide by 0.85 to get the answer:

\[
P = \frac{70}{0.85} = 82.35
\]

The correct option is then (c).
13) **Answer:** b

**Solution:** You need to take the average number of rabies cases weighted by the percentage of cities. The weighted average calculation is:

\[
\frac{0.4 \times 0 + 0.25 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.05 \times 6}{0.4 + 0.25 + 0.2 + 0.1 + 0.05} = \frac{1.8}{1} = 1.8
\]

So there are an average of 1.8 cases of rabies per city: option (b) is correct.
14) **Answer:** c, d

**Solution:** Two cities have the same population exactly when their lines intersect. Lines which are parallel to each other -- that is to say, have the same slope -- will never cross. This is true for cities **B and C (option c)**.

Furthermore, if one city starts out larger than another one and is growing more quickly, the two will never have the same population. This occurs when the city whose line starts out higher up on the population axis is also steeper (has a greater slope). This is true for cities **B and D (option d)**.

The other pairs of cities all have the same population at some point: A crosses the lines for B, C, and D, and C and D start out with the same population.
15) **Answer:** 8 meters

**Solution:** You need to know \( r \), the length of rope between the boat and the jetty, which can be calculated using the Pythagorean theorem:

\[
\begin{align*}
    r^2 &= 14^2 + 6^2 = 232 \\
    r &= \sqrt{232} \approx 15.23 \text{ m}
\end{align*}
\]

When you drive forward 5 meters, the rope stretching between the boat and the jetty becomes five meter shorter, so is 10.23 meters. The jetty is still 6 meters tall, but \( d \), the distance from the boat to the jetty, has changed. Now, you use the Pythagorean theorem again, to find \( d \):

\[
\begin{align*}
    d^2 + 6^2 &= 10.23^2 \\
    d^2 &= 10.23^2 - 6^2 = 68.65 \\
    d &= \sqrt{68.65} \approx 8.3
\end{align*}
\]

The distance between the boat and the base of the jetty is approximately 8 meters.