Measuring the Mean Decay Lifetime of Positive and Negative Muons

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Muons are leptons formed high above the Earth by the interaction of cosmic rays with the constituents of our atmosphere. They have a mean vacuum lifetime of $2.1969811 \pm 0.0000022 \mu$s. Using a scintillation detector in an above-ground laboratory, I have successfully recorded the passage and decays of both positive and negative muons. My experimental value for the decay lifetime of the combined muon population, $2.143 \pm 0.005 \mu$s, is consistent with the mean expected lifetime of muons in the detector, $2.15 \pm 0.01 \mu$s, which depends upon the relative populations of the two types of muons and the capture interaction that negative muons can have with the detector material.

BACKGROUND: THE MAKING OF MUONS

At the top of the troposphere, 20 km above the surface of the Earth, there is a continuous flux of high energy charged particles from other parts of the universe. This flux is approximately 2% electrons and 98% heavier nuclei, of which about 87% are protons, 12% are alpha particles, and the remainder are even heavier nuclei that are produced by stellar nucleosynthesis [1].

This flux of charged particles collides with the nuclei of nitrogen and oxygen molecules in the atmosphere to produce a shower of elementary particles. Those elementary particles undergo various electromagnetic and nuclear interactions to produce even more particles in a cascade. Of these secondary particles, the charged pions are the primary source of muons. Pions will decay spontaneously into a muon plus a neutrino or antineutrino:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu,$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu.$$  \hspace{1cm} (1)

Negative pions produce negative muons and positive pions produce positive muons [1]. Because the negative pion can decay in more than one way, slightly fewer negative muons than positive muons will be produced [2].

Both positive and negative muons can interact with matter through the weak and electromagnetic forces. They travel long distances through the atmosphere, losing approximately 2 GeV of energy due to electromagnetic interaction, until they finally decay via the weak force:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu,$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu.$$  \hspace{1cm} (2)

This decay produces an electron or a positron, a neutrino, and an antineutrino [1]. Negative muons produce electrons and positive muons produce positrons.

A given initial population of muons, $N_0$, will decay over time and the number of muons left at time $t$ is given by:

$$N(t) = N_0 e^{-\lambda t},$$
$$\ln(N(t)) = \ln(N_0) - \lambda t,$$  \hspace{1cm} (3)

where $\lambda$ is the decay parameter and $\tau = 1/\lambda$ is the mean lifetime. The mean lifetime of both positive and negative muons in vacuum is $2.1969811 \pm 0.0000022 \mu$s [3], which is the average length of time a muon exists before it undergoes decay. A related concept, the half life ($t_{1/2} = \tau \ln(2)$) is the amount of time that must pass before half the muons in a given sample have decayed.

MUON DETECTION BASICS

To measure the muon decay lifetime, a positive or negative muon must enter the detector, lose enough kinetic energy that it stops inside the detector, and then decay. The measured lifetime is recorded as the time interval between the muon coming to rest and the muon decaying.

The main detector body is made up of a plastic scintillator and a photomultiplier tube. A scintillator is any type of material that emits light when excited, such as when a muon enters or decays. A photomultiplier tube (PMT), which is a highly sensitive detector of photons, uses high voltage to amplify the light signal from the scintillator into a detectable pulse of electrons. These electrons then pass as a current into the measuring devices that make up the rest of the experimental setup.

Some of the amplified light signals from the PMT will be muons, and the height of those signal pulses will depend most strongly upon the energy of the muon. Some will be “dark”, meaning that they are spontaneous pulses that originate inside the PMT itself and are not due to the detection of light in the scintillator. While small in amplitude, these “dark” pulses represent a significant source of noise. For this reason, the pulses from the PMT first pass through a discriminator. The discriminator has an adjustable threshold such that only pulses with a peak height exceeding the threshold are accepted.

The discriminator uses simple boolean logic to decide what happens to each incoming pulse. An accepted pulse (logic 1) results in a -0.8 V output pulse that is approximately 5 ns in duration. A rejected pulse (logic 0) results in a 0 V output pulse.

The pulses from the discriminator are then used as
inputs to a Time to Amplitude Converter (TAC). The TAC accepts a “start” pulse and waits for a second pulse, known as the “stop” pulse, within a configurable time window. If no “stop” pulse is detected, the TAC resets and waits for another “start” pulse. If a “stop” pulse is received, then the difference in time between the “start” and “stop” pulses is turned into an output pulse with an amplitude between 0 and 10 V. The height of the output pulse is directly proportional to the time difference between the “start” and “stop” pulses. The TAC is used to detect the time interval between pairs of pulses that occur when a muon comes to rest inside the detector and when that muon decays.

From the TAC, the output pulses are sent to a multichannel analyzer (MCA). The MCA analyzes the pulses and places them into a histogram according to their amplitude. The muon lifetime can then be extracted from the data in this histogram [2].

PREDICTING THE EXPECTED LIFETIME

The mean lifetime of positive muons, whether in vacuum or passing through matter, is constant. Negative muons, however, can be captured by a nucleus much like an electron, resulting in a muonic atom that can decay in two ways. The first way is via ordinary muon decay (see Equation 2). The second way changes a proton in the nucleus to a neutron with the emission of a muon neutrino. This second form of decay has a shorter lifetime, with the result that the mean lifetime of negative muons in matter is slightly less than the mean lifetime of negative muons in vacuum [1].

I can predict the mean lifetime of all muons in the detector by first estimating the mean lifetime of negative muons in the detector, then using that estimated lifetime along with the ratio of positive to negative muons at sea level to arrive at a predicted mean lifetime for the combined population of muons.

To estimate the lifetime of negative muons in the detector, I need to know the lifetime of negative muons in the material of the scintillator. According to the specification sheet [4], the EJ-200 scintillator is made up of polyvinyltoluene with the characteristics listed in Table I. From those specifications, the relative percentages of hydrogen and carbon atoms can be calculated.

\[
\text{Z eff} = \sqrt[4]{\left(\frac{\% H}{100}\right) \cdot Z_{eff,H}^4 + \left(\frac{\% C}{100}\right) \cdot Z_{eff,C}^4}
\]

\(Z\) is the atomic number of the capturing nucleus [1]. Since the scintillator is made up of both hydrogen and carbon atoms, an effective \(Z\) value for the scintillator material must be calculated.

As shown in Table II, the effective \(Z\) values of hydrogen and carbon are 1.000 and 5.673, respectively [5]. These values represent the effective positive charge seen by a valence electron due to the shielding effects of other electrons. When combined with the estimated proportions of hydrogen and carbon from Table I, the effective \(Z\) value for the scintillator material as a whole was found to be:

\[Z_{eff} = 4.71.\]

This is very close to the effective \(Z\) value of \(^{10}\text{B}\), which is listed in Table II as 4.680 [5] with an associated negative muon lifetime of \(2.082 \pm 0.006 \mu\text{s} [6]\). This value was therefore chosen as the estimated negative muon lifetime in the scintillator. I took the known vacuum lifetime value for muons as the positive muon lifetime.

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The ratio of positive to negative muons at sea level has been measured by the Compact Muon Solenoid (CMS) experiment to be \(1.2766 \pm 0.0064\), and is valid for muons with momentum less than 100 GeV/c [7]. Using this ratio, it is possible to estimate the relative percentages of positive and negative muons passing through the detector, which are listed in Table III along with their expected lifetimes in the material of the scintillator.

\[
\tau_{adj} = \frac{\% +}{100} \cdot \tau_+ + \frac{\% -}{100} \cdot \tau_-
\]

\(\tau_{adj}\) is the predicted mean lifetime of muons in the detector.

\[
\tau = 2.15 \pm 0.01 \mu\text{s}.
\]
FIG. 1. A simplified schematic of the experimental setup, showing the relevant connections and direction of signal pulses. Not shown are the frequency counter and oscilloscope I used to calibrate and adjust the equipment.

EXPERIMENTAL SETUP

The scintillator used was an Eljen Technology EJ-200, which is a cylinder 11 inches in diameter and 11 inches tall, sealed inside a light-tight aluminum casing. There are two Burle 8575 photomultiplier tubes mounted at the base, though only one was in use. The PMT was connected to a Glassman EL Series high voltage power supply, which was set to -1750 V.

From the PMT, a 50 Ohm BNC cable was connected to the front input of an Ortec 584 discriminator. While testing the output of the discriminator with a frequency counter, the discriminator threshold was adjusted to -185 mV. This value represented the point at which “dark” pulses were almost entirely rejected, while muon pulses occurring approximately once every 2 seconds (1/2 Hz) were accepted.

A 50 Ohm BNC cable was then used to connect the output of the discriminator to the “start” and “stop” inputs of the TAC. This was accomplished using a tee and a coiled, 5 meter length of cable. One output of the tee was connected directly to the “stop” input on the TAC. The other output of the tee was connected to the 5 meter length of cable before being connected to the “start” input. (See Fig. 1). The purpose of this extra length of cable was to create a 25 ns delay between the “start” and “stop” signals. Without this delay, each pulse that entered the TAC would have simultaneously triggered the “start” and “stop” signals, resulting in a recorded time difference of zero for every pulse. The TAC was set to a range of 200 ns with a multiplier of 100, corresponding to a maximum time difference between pulse pairs of 20 µs.

The TAC output was then connected with a 50 Ohm BNC cable to the input of the MCA, and the MCA was connected via USB cable to the PC. Using the provided software, the MCA was configured to 1024 channels, with a range of 0 to 10 V to correspond to the maximum possible pulse height from the TAC. The live time was set to 720,000 seconds and data acquisition was started.

DATA ANALYSIS

The experiment ran twice for a total of 1,206,926 seconds of live time and counted 22,231 events (Fig. 2).

To calibrate the time base of the MCA, the output of a function generator was connected to the discriminator, and pulse pairs with a known time difference between them, simulating muon decay events, were fed through the discriminator to the MCA. The output channel on the MCA was then recorded. A linear least squares fit of time difference versus MCA channel was performed on these data points, assigning each point an equal weight. The slope of the resulting fitted line was taken to be the amount of time (Δt) represented by each MCA channel, while the square root of the diagonal of the associated covariance matrix element was taken to be the error of the slope. This yielded a Δt of 19.60 ± 0.04 ns.

Using that Δt, I was then able to convert the x axis of the MCA data from channel number to a relative elapsed time in microseconds by multiplying each MCA channel number by Δt.

FIG. 2. The raw data from the MCA, with the calibrated time base substituted on the x axis. Counts shaded in red were cropped from the fit. Counts shaded in gray were used for the background estimate.

Of note in the raw data is that the exponential distribution never decays to zero, as would be expected. This is due to the presence of uniformly-distributed background events which happened in the scintillator close enough together to be counted by the TAC as a muon stopping and decaying, but may have been unrelated.
By taking the natural log of the data, it is possible to see that there is a point at which the downward slope transitions into a constant background (see Figure 2). The first step in the data analysis was to estimate this background and subtract it so that I could use Equation 3 for the fit. For that purpose, I truncated the data at 10.66 \( \mu s \), which is located at that important transition point. The data from 10.66 \( \mu s \) to 20.0 \( \mu s \) was averaged, yielding an estimated background of 1.49 counts per data point, which was then subtracted from all data points.

Also of note is the presence of two anomalous peaks in the data, the second of which appears around 0.5 \( \mu s \). When muons pass through the detector, a molecule of residual gas can be ionized by the flow of electrons through the PMT. This ionized molecule will then travel slowly towards the photocathode, striking a dynode or the photocathode itself, creating a secondary wave of electrons known as an “after pulse” [2]. Since these “after pulses” would have had a significant effect on the curve fit, all data prior to 0.5 \( \mu s \) was excluded from the fit.

Finally, since the fit was to be obtained by taking the natural log of the data and fitting a straight line to it using weighted linear least squares, it was necessary to remove any data points that were less than 1 after background subtraction. The resulting fit range was therefore from 0.5 \( \mu s \) to 7.76 \( \mu s \).

![Linear Fit](image)

**FIG. 3.** The weighted linear least squares fit to the natural log of the data, with error bars on each data point.

Figure 3 shows the results of the weighted linear least squares fit. The slope of this fit line is the decay parameter \( \lambda \) from Equation 3, and the associated error is the error for \( \lambda \) as reported by the fit via the covariance matrix. The mean muon lifetime \( \tau \) is the inverse of the decay parameter, and was determined to be 2.143 ± 0.005 \( \mu s \), which is 10.8 sigma lower than the vacuum value of 2.1969811 ± 0.0000022 \( \mu s \) [3], but within 0.717 sigma of the predicted lifetime value of 2.15 ± 0.01 \( \mu s \).

**CONCLUSION**

I was successfully able to record muon decays using a scintillation detector in an above-ground laboratory. I was then able to extract a muon lifetime of 2.143 ± 0.005 \( \mu s \) using a weighted linear least squares fit. My experimental lifetime value is much lower than the accepted vacuum lifetime of 2.1969811 ± 0.0000022 \( \mu s \) [3], but consistent with my predicted muon lifetime of 2.15 ± 0.01 \( \mu s \) within the material of the detector.

| \( \tau \) (\( \mu s \)) |   
|-----------------|-----------------|
| Vacuum          | 2.1969811 ± 0.0000022 |
| Predicted       | 2.15 ± 0.01 |
| Experimental    | 2.143 ± 0.005 |

**TABLE IV.** A comparison of the the known vacuum lifetime [3], the predicted lifetime in the detector (Equation 5), and the experimentally measured lifetime.

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